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Introduction to Set Theory

1. Basic concepts.

(1) A SET is a collection of objects.

(2) An object is an ELEMENT OF a set A if that object is a member of the collection A.
Notation: "$\in$" reads as "is an element of" or "belongs to".

(3) A set A is a SUBSET OF a set B if all the elements of A are also in B.
Notation: "$\subseteq$" reads as "is a subset of".

(4) The INTERSECTION of two sets A and B ($A \cap B$) is the set containing all and only the objects that are elements of both A and B.

(5) The UNION of two sets A and B ($A \cup B$) is the set containing all and only the objects that are elements of A, of B, or of both A and B.

(6) The COMPLEMENT of a set A (A, or A’) is the set containing all the individuals in the discourse except for the elements of A.

(7) The POWER SET of a set A ($\wp$(A)) is the set whose members are all the subsets of A.

QUESTION 1: Given the sets under (8) and assuming that the universe of the discourse is $\cup$\{A, B, C, D, E, F, G\}, list the members of the following sets:

(8)

A= {1, 2, 3, 4}	E = { {1}, 2, {a, 1} }
B = {a, b, c, d, e, f}	F = {1, c, d}
C = {1, 2}	G = {d, e, 2, 3}
D = {1, 3, 4, a, b}	

(9)
a. C-D =
b. $A \cap F$ =
c. $A \cap B$=
d. $C' \cap F'$ =
e. $E \cap C$ =
f. $(C \cup D) - (C \cap D)$ =
g. $F \cup C$ =
h. $G' \cap C$ =
i. $A \cap E$ =
j. $(E \cup B) \cup D$ =

2. Relations.

■ Ordered Pairs and Cartesian Product:

(10) Ordered pair/n-tuple: a set with n-elements where order matters.
$\langle a,b\rangle =_{def} \{\{a\},\{a,b\}\}$

(11) Cartesian Product:
$A\times B =_{def} \{\langle x,y\rangle \mid x\in A \text{ and } y\in B\}$

■ Relations: a relation is a set of pairs (or, more generally, of n-tuples). E.g., "mother of", "to be sitting to the right of". Relation in A. E.g. "advisor of". Relation from A to B.

(12) A *relation* from A to B is a subset of $A\times B$.
A *relation* in A is a subset of $A\times A$.

(13) a. Domain of R: $\{a \mid \text{there is some } b \text{ such that } \langle a,b\rangle \in R\}$
b. Range of R: $\{b \mid \text{there is some } a \text{ such that } \langle a,b\rangle \in R\}$

(14) Complement of a relation R from A to B (a relation $R\subseteq A\times B$): R'
$R' =_{def} (A\times B) - R$

QUESTION 2: Take $A=\{a,b,c\}$, $B=\{1,2,3\}$ and $C=\{a,b\}$. $R=\{\langle a,1\rangle, \langle b,2\rangle, \langle b,3\rangle\}$.
What is the complement of the relation R from A to B?
What is the complement of the relation R from C to B?

(15) Inverse of a relation: R^{-1}
$R^{-1} =_{def} \{\langle b,a\rangle \mid \langle a,b\rangle \in R\}$

3. Funtions.

■ Functions:

(16) A relation R from A to B is a *function* from A to B ($F{:}A\rightarrow B$) iff:
a. The domain of R is A. (except for partial "functions")
⇒ Every member of A appears at least once as first member of a pair.
b. Each element in the domain is paired just with one element in the range
⇒ Every member of A appears at most once as first member of a pair.

(17) Argument and value: $F(a) = b$

(18) a. Functions from A to B are called functions *into* B.
b. Functions from A to B such that the range of the function equals B are called *onto* B.

(19) a. Functions from A to B where every member of B is assigned at most once to a member of A are called *one-to-one*.
b. Otherwise, we'll call them *many-to-one*.

(20) Functions that are at the same time onto and one-to-one are *called one-to-one correspondences*.

■ The characteristic function of a set:

(21) a. Let A be a set. The, $char_A$, the *characteristic function of A*, is the function F such that,

for any $x \in A$, $F(x)=1$, and

for any $x \notin A$, $F(x)=0$.

b. Let F be a function with range $\{0,1\}$. Then, $char_F$, the *set characterized by F*, is

$\{x \in D \mid F(x)=1\}$

■ Schönfinkelization:

(22) $\mathbf{U} = \{a,b,c\}$

(23) The relation "fond of":

$R_{fond\text{-}of} = \{<a,b>, <b,c>, <c,c>\}$

(24) The characteristic function of $R_{fond\text{-}of}$:

$Char_{Rfond\text{-}of}$ =

$<a,a>$	$\rightarrow$	0
$<a,b>$	$\rightarrow$	1
$<a,c>$	$\rightarrow$	0
$<b,a>$	$\rightarrow$	0
$<b,b>$	$\rightarrow$	0
$<b,c>$	$\rightarrow$	1
$<c,a>$	$\rightarrow$	0
$<c,b>$	$\rightarrow$	0
$<c,c>$	$\rightarrow$	1

(25) Turning n-ary functions into multiple embedded 1-ary functions: Schönfinkelization.

a. Left-to-right:

		a	$\rightarrow$	0
a	$\rightarrow$	b	$\rightarrow$	1
		c	$\rightarrow$	0
		a	$\rightarrow$	0
b	$\rightarrow$	b	$\rightarrow$	0
		c	$\rightarrow$	1
		a	$\rightarrow$	0
c	$\rightarrow$	b	$\rightarrow$	0
		c	$\rightarrow$	1

b. Right-to-left:(inverse + left-to-r. shonf.)

		a	$\rightarrow$	0
a	$\rightarrow$	b	$\rightarrow$	0
		c	$\rightarrow$	0
		a	$\rightarrow$	1
b	$\rightarrow$	b	$\rightarrow$	0
		c	$\rightarrow$	0
		a	$\rightarrow$	0
c	$\rightarrow$	b	$\rightarrow$	1
		c	$\rightarrow$	1

EXERCISE 1: Given the following Universe, spell out the characteristic function of the set in (27) and schönfinkelize it right-to-left.

(26) **U** = {d, e}

(27) R = { <d,d,d>, <d,e,d>, <e,d,d>, <e,e,e>, <e,e,d> }