Assignment 5
Ling 106
Due April 1, 2002 in class

1. Do exercises 1.1 and 1.2 from Sipser.

2. Construct state diagrams for deterministic finite state automata (DFA) that accept each of the languages below using the fewest states possible.

   Make sure you write valid automata: indicate the start state and all accept states, and check that there are no missing or duplicate arrows. (Writing the transition function helps with this.) Then check that your automaton does what the problem asks. Remember that your automaton must recognize all the strings in the language and no other strings.

   (a) Alphabet: \{a, b\}. L = the set of all strings of even length.
      (Note: Zero is an even number.)
   
   (b) Alphabet: \{a, b\}. L = \{w \mid w \text{ contains exactly one } b\}.
   
   (c) Alphabet: \{0, 1\}. L = \{w \mid w \text{ contains at least three } 1\text{s}\}.
   
   (d) Alphabet: \{a, b\}. L = \emptyset.

3. Construct a DFA that accepts the union of the language accepted by \(M_1\) and the language accepted by \(M_2\). Give a formal description of the new machine.

   The formal description of \(M_1\):
   \[
   (\{q_1, q_2, q_3\}, \{a, b\}, \delta, q_1, \{q_2\}), \text{ where } \delta \text{ is defined as:}
   \]
   \[
   \begin{array}{c|cc}
   \delta & a & b \\
   \hline
   q_1 & q_2 & q_1 \\
   q_2 & q_3 & q_3 \\
   q_3 & q_2 & q_1 \\
   \end{array}
   \]

   The formal description of \(M_2\):
   \[
   (\{q_1, q_2, q_3, q_4\}, \{a, b\}, \delta, q_1, \{q_3, q_4\}), \text{ where } \delta \text{ is defined as:}
   \]
   \[
   \begin{array}{c|cc}
   \delta & a & b \\
   \hline
   q_1 & q_1 & q_2 \\
   q_2 & q_3 & q_4 \\
   q_3 & q_2 & q_1 \\
   q_4 & q_3 & q_4 \\
   \end{array}
   \]

4. Let \(A = \{a, ab\}\), \(B = \{b, ba, bb\}\). Give:
   a. \(A \cup B\)
   b. \(A \cap B\)
   c. \(B^*\)
   d. \(A \circ B\)

   (You should give all elements of each set, unless it is infinite).
5. For \( B = \{b, ba, bb\} \) as above, which of the following are in \( B^* \)?

   a. baba  
   b. babbab  
   c. bcde  
   d. bbbbab  
   e. \( \varepsilon \)  
   f. abbbb  
   g. bab  
   h. b  
   i. baaba

6. Do Exercise 1.15 from Sipser (problem a. is given here as an example). In your homework, please write out each regular expression and include it along with your answers. Answers must take the following form:
   a. \( a^*b^* \)
      contains: \( ab, abb \)
      does not contain: \( ba, aba \)

7. a. Give a regular expression that generates the set of all strings containing a total of exactly two 1s (i.e., any number of 0s is allowed).
   b. Give a regular expression that generates any string containing the substring 010 anywhere within it.

8. Consider the following state diagram, where \( q0 \) is the start state:

![State Diagram]

Convert this FSA to a right linear grammar (give a formal description of the grammar).