Assignment 3 Answer Key
Ling 106

1. (a) The sentence “It is not the case that the stained glass has been ordered and the Browns have already paid for it” is ambiguous. Translate each one of the two readings into a formula of PL.

   Let $p = \text{“The stained glass has been ordered.”}$
   Let $q = \text{“The Browns have paid for it.”}$

   The two readings are $\neg(p \land q)$ and $(\neg p \land q)$.

(b) There is one situation in which both readings are true—what is it?

   $\llbracket \neg(p \land q) \rrbracket = \llbracket (\neg p \land q) \rrbracket = 1$ iff $\llbracket p \rrbracket = 0$ and $\llbracket q \rrbracket = 1$.

   That is, both are true just in case $p$ is false and $q$ is true.

(c) There is one situation in which both readings are false—what is it?

   $\llbracket \neg(p \land q) \rrbracket = \llbracket (\neg p \land q) \rrbracket = 0$ iff $\llbracket p \rrbracket = \llbracket q \rrbracket = 1$.

   That is, both are false just in case $p$ is true and $q$ is true.

The same facts are demonstrated in the following truth table:

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
<th>$\neg(p \land q)$</th>
<th>$\neg p$</th>
<th>$(\neg p \land q)$</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

2. Consider the following undirected graph $G = (V, E)$, where $V = \{a, b, c, d, e, f\}$ and $E = \{\{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, e\}, \{c, e\}, \{c, d\}, \{d, e\}\}$.

(a) Is the graph connected? No

   Why or why not? There is no path between node $f$ and node $d$ (for example).

(b) Give a sequence of edges that corresponds to a simple path in the graph.

   One of the simple paths is $\{(a, b), (b, e), (d, e)\}$.

(c) Is there a simple cycle in the graph? Yes

   If so, give a sequence of nodes that corresponds to a simple cycle in the graph.

   One of the simple cycles is $(a, d, c, b, a)$. 

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3. Consider the following directed graph $T = (V, E)$, where $V = \{a, b, c, d, e, f\}$ and $E = \{(a, b), (a, c), (b, d), (b, e), (d, f)\}$.

(a) Is the graph connected? \hspace{1cm} \textbf{Yes}

(b) Give a sequence of edges that corresponds to the longest directed path in the graph.

\[ ((a, b), (b, d), (d, f)) \]

c) Is there a simple cycle in the graph? \hspace{1cm} \textbf{No}

4. Complete the following sentence by choosing the correct term from each of the underlined pairs:

(a) A tree is a \textbf{connected, directed} graph with \textbf{no simple cycles}.

(b) Is the following graph $S$ a tree? $S = (V, E)$, where $V = \{a, b\}$ and $E = \{(a, b)\}$. \hspace{1cm} \textbf{Yes}

5. Let the alphabet $A = \{a\}$.

Let the language $L \subseteq A^\ast = \{w : |w| \leq 5\}$.

(a) List the members of $A^\ast$.

\[ A^\ast = \{\epsilon, a, aa, aaa, aaaa, aaaaa\} \]

(b) List the members of $L$.

\[ L = \{\epsilon, a, aa, aaa, aaaa\} \]

6. Let the alphabet $\Sigma = \{a, b, c, d\}$.

Let the language $L \subseteq \Sigma^\ast = \{w : |w| \leq 3 \text{ and } ab \text{ is a substring of } w\}$.

(a) List the members of $L$.

\[ L = \{ab, aab, aba, abb, abc, abd, bab, cab, dab\} \]

(b) What is $|L|$?

$|L| = 9$

7. Let the string $s = \text{dcb}$ and let the string $t = \text{cba}$.

(a) $s^R = \text{bcd}$

(b) $t^3 = \text{cbacbaca}$

(c) $(s^R)^2 = \text{bcbdcdbcd}$

(d) $(s^2)^R = \text{bcbdcdbcd}$

(e) $t \cdot s = \text{cbadcb}$

(f) $s^2 \cdot t^R = \text{dcbabc}$

(g) $(s \cdot s)^t = \text{dcbdcbca}$

(h) $s^2 \cdot (s \cdot t) = \text{dcbdcbca}$

8. \textbf{Extra credit.} Concatenation of languages $L_1$ and $L_2$ (i.e., $L_1 \cdot L_2$) is defined as follows: $L_1 \cdot L_2 = \{s \cdot s^2 : s^1 \in L_1 \text{ and } s^2 \in L_2\}$. If $L_1 = \{a, b, ab\}$ and $L_2 = \{c, d\}$, what is $L_1 \cdot L_2$?

\[ L_1 \cdot L_2 = \{ac, ad, bc, bd, abc, abd\} \]