Assignment 3
Ling 106
Due February 18, 2002 in class

1. Suppose I say “It is not the case that the stained glass has been ordered and the Browns have already paid for it”.
   (a) The sentence in quotation marks above is ambiguous. Translate each one of the two readings into a formula of PL. (Remember to provide a key for interpreting the formulae, e.g., p="the stained glass has been ordered").
   (b) There is one situation in which both readings are true—what is it?
   (c) There is one situation in which both readings are false—what is it?
   [Hint: Constructing truth tables for your two expressions of PL will help you to answer (b) and (c).]

2. Consider the following undirected graph $G$:
   $G = (V, E)$, where
   $V = \{a, b, c, d, e, f\}$
   $E = \{(a, b), (a, c), (a, d), (b, c), (b, e), (c, e), (c, d), (d, e)\}$
   [Note: It might be helpful for you to draw the graphs before answering the questions, but it is not necessary to include the drawings on your homework.]
   (a) Is the graph connected? Why or why not?
   (b) Give a sequence of edges that corresponds to a simple path in the graph.
   (Recall that a sequence is an ordered set.)
   (c) Is there a simple cycle in the graph? If so, give a sequence of nodes that corresponds to a simple cycle in the graph.

3. Consider the following directed graph $T$:
   $T = (V, E)$, where
   $V = \{a, b, c, d, e, f\}$
   $E = \{(a, b), (a, c), (b, d), (b, e), (c, d), (d, f)\}$
   (a) Is the graph connected? Why or why not?
   (b) Give a sequence of edges that corresponds to the longest directed path in the graph.
   (c) Is there a simple cycle in the graph? If so, give a sequence of nodes that corresponds to a simple cycle in the graph.
   (d) What node is the root?
   (e) What node(s) are leaves?
   (f) What pair(s) of nodes are sisters?
   (g) What is the height of the tree?
   (h) What is the level of node $a$?
   (i) What is the level of node $e$?
   (j) What is the indegree of node $a$?
   (k) What is the outdegree of node $b$?
4. Complete the following sentence by choosing the correct term from each of the underlined pairs:
   (a) A tree is a(n) \{connected/unconnected\}, \{directed/undirected\} graph with \{simple cycles/no simple cycles\}.
   
   (b) Is the following graph \(S\) a tree?  
   \(S = (V, E)\), where  
   \(V = \{a, b\}\)  
   \(E = \{(a, b)\}\)

5. Let the alphabet \(A = \{a\}\).  
   Let the language \(L \subseteq A^* = \{w: |w| \leq 5\}\).
   (a) List the members of \(A^*\).
   [Note: Recall that \(A^*\) is the set of all strings on \(A\). Since the set is infinite, you will have to use “...” at some point in your list for (a). That is the only place in this assignment where it is appropriate for you to use ”...”.
   (b) List the members of \(L\).

6. Let the alphabet \(\Sigma = \{a, b, c, d\}\).  
   Let the language \(L \subseteq \Sigma^* = \{w: |w| \leq 3 \text{ and } ab \text{ is a substring of } w\}\).
   (a) List the members of \(L\).
   (b) What is \(|L|\)?

7. Let the string \(s = \text{dcb}\) and let the string \(t = \text{cba}\).
   (a) \(s^R = \)
   (b) \(t^3 = \)
   (c) \((s^R)^2 = \)
   (d) \((s^2)^R = \)
   (e) \(t^{-}s = \)
   (f) \(s^{-}t^R = \)
   (g) \(s^{-}s^{-}t = \)
   (h) \(s^{-}(s^{-}t) = \)

8. Extra credit. Concatenation of languages \(L_1\) and \(L_2\) (i.e., \(L_1 \sqcup L_2\)) is defined as follows: \(L_1 \sqcup L_2 = \{s = s^1s^2 | s^1 \in L_1 \text{ and } s^2 \in L_2\}\)
   If \(L_1 = \{a, b, ab\}\) and \(L_2 = \{c, d\}\), what is \(L_1 \sqcup L_2\)? (List the members of this set.)