Assignment 1
Ling 106
Due January 28, 2002 in class

1. Given the following four sets:
   
   \[ A = \{3, \text{Lucy}\} \]
   \[ B = \{\{3\}, \{\text{Lucy}\}\} \]
   \[ C = \{\{3\}, \{3, \text{Lucy}\}\} \]
   \[ D = \{\{3\}, \{\text{Lucy}\}, \{3, \text{Lucy}\}\} \]

   Which of the following statements are true?

   (a) \[ A = B \]
   (f) \[ A \in C \]
   (k) \[ B \subseteq D \]
   (o) \[ A \subseteq C \]
   (b) \[ A \subseteq B \]
   (g) \[ A \subseteq D \]
   (l) \[ B \in D \]
   (p) \[ B \subseteq C \]
   (c) \[ D \cup C = D \]
   (h) \[ D \cap C = C \]
   (m) \[ A \cap \emptyset = A \]
   (q) \[ A \cup \emptyset = A \]
   (d) \[ C - D = \emptyset \]
   (i) \[ D - C = \{\{\text{Lucy}\}\} \]
   (n) \[ \emptyset - A = \emptyset \]
   (r) \[ C \subseteq D \]
   (e) \[ A \in D \]
   (j) \[ A - \emptyset = A \]

2. Let \( S \) be \( \{x, y, z\} \).

   (a) What is the Cartesian product of \( S \) with itself? \( S \times S = \ldots \)
   (b) Is \( S \subseteq S \times S \)?
   (c) Is \( S \times S \subseteq S \)?
   (d) Is \( |S \times S| > |S| \)?
   (e) What is the power set of \( S \)? \( \mathcal{P}(S) = \ldots \)
   (f) Is \( S \subseteq \mathcal{P}(S) \)?
   (g) Is \( \mathcal{P}(S) \subseteq S \)?
   (h) Is \( |S| > |\mathcal{P}(S)| \)?

3. Note: To reduce clerical error, keep the sets you define here small.

   (a) Define a set \( A \). Let \( A = \ldots \)
   (b) Define a set \( B \). Let \( B = \ldots \)
   (c) What is the Cartesian product of \( A \) and \( B \)? \( A \times B = \ldots \)
   (d) Define a relation \( R \) in \( A \times B \). Let \( R \subseteq A \times B = \ldots \)
   (e) What is \( R^t \)? \( R^t = \ldots \)
   (f) What is \( R^{-1} \)? \( R^{-1} = \ldots \)
   (g) Is the relation that you defined in (d) a function?

   If yes: (h) Why (i.e., how can you tell)?
   (i) Define the function. \( f : A \to B = \ldots \)
   If not: (j) Why (i.e., how can you tell)?
   (k) Define a function in \( A \times B \). \( f : A \to B = \ldots \)
4. Let $X$ be the set \{1, 2, 3, 4, 5\} and $Y$ be the set \{6, 7, 8, 9, 10\}. The unary function $f : X \rightarrow Y$ and the binary function $g : X \times Y \rightarrow Y$ are described in the following tables.

| $n$ | $f(n)$ | \[ \begin{array}{l} \hline 1 & 6 & \hline 2 & 7 & \hline 3 & 6 & \hline 4 & 7 & \hline 5 & 6 & \hline \end{array} \] | \[ \begin{array}{l} \hline 1 & 10 & \hline 2 & 7 & \hline 3 & 7 & \hline 4 & 9 & \hline 5 & 6 & \hline \end{array} \] | \[ \begin{array}{l} \hline 6 & 10 & \hline 7 & 9 & \hline 8 & 8 & \hline 9 & 7 & \hline 10 & 6 & \hline \end{array} \] |

(a) What is the value of $f(2)$?
(b) What is the domain of $f$?
(c) What is the range of $f$?
(d) What is the value of $g(2, 10)$?
(e) What is the domain of $g$?
(f) What is the range of $g$?
(g) What is the value of $g(4, f(4))$?
(h) Is $f$ "onto"?
(i) Is $g$ "onto"?
(j) Is $f$ "one-to-one"?
(k) Is $g$ "one-to-one"?

5. Give the name of the set-theoretic equality demonstrated by each of the following (‘$U$’ is the universe):

(a) $(B')' = B$
(b) $M \cup M = M$
(c) $K \cup U = U$
(d) $K - L = K \cap L'$
(e) $M \cup N = N \cup M$
(f) $(B \cup C)' = B' \cap C'$
(g) $K \cap \emptyset = \emptyset$
(h) $(M \cap N) \cap P = M \cap (N \cup P)$
6. Give the name of the law of propositional logic demonstrated by each of the following ('T' is a tautology and 'F' is a contradiction; the symbols φ, ψ, and Γ stand for any statement of propositional logic whether atomic or complex):

(a) \((φ \land ψ) \land Γ\) ⇔ \((φ \land (ψ \lor Γ))\)
(b) \(¬(φ \lor ψ) ⇔ (¬φ \land ¬ψ)\)
(c) \(¬¬φ ⇔ φ\)
(d) \((φ \lor F) ⇔ φ\)
(e) \((φ → Γ) ⇔ (¬Γ → ¬φ)\)
(f) \((φ \land T) ⇔ φ\)
(g) \((φ \lor φ) ⇔ φ\)
(h) \((Γ \lor ψ) ⇔ (Γ \lor φ)\)

7. For each of the following proofs, is the proof valid? That is, does each transition correspond to a law of equality or a rule of inference (see pages 18, 110, and 117 of Partee)? If not, identify the error(s) in the proof.

(a) Proof I:
1. \(A \cap (B - A)\)
2. \(A \cap (B \cup A')\)
3. \((B \cup A') \cap A\)
4. \(B \cup (A' \cap A)\)
5. \(B \cup (A \cap A')\)
6. \(B \cup \emptyset\)
7. \(\emptyset\)

(b) Proof II: (Recall that T is a tautology.)
1. \(¬p \lor (p \land q)\)
2. \((¬p \lor p) \land (¬p \lor q)\)
3. \((p \lor ¬p) \land (¬p \lor q)\)
4. \(T \land (¬p \lor q)\)
5. \((¬p \lor q) \land T\)
6. \(¬p \lor q\)
7. \(q → p\)

(c) Proof III:
1. \(p ⇔ q\)
2. \((p → q) \land (q → p)\)
3. \((q → p) \land (p → q)\)
4. \(q → p\)
5. \(¬q \lor p\)

3
8. **Extra credit 1**: Write formal descriptions of the following sets using the method of specification noted.
   (a) The set containing the numbers 200, 2, and 20. [list]
   (b) The set containing all natural numbers greater than 5. [recursive rules]
   (c) The set containing all natural numbers less than or equal to 5. [predicate notation]
   (d) The set containing the empty set. [list]
   (e) The set containing nothing at all. [list]
   (f) The set containing all of your matrilineal ancestors. [recursive rules]

9. **Extra credit 2**: Answer the questions and explain your answers (you may do this by constructing examples).
   (a) If \( x \subseteq E \) and \( E \subseteq F \), is it necessarily true that \( x \subseteq F \)?
   (b) If \( x \in E \) and \( E \in F \), is it necessarily true that \( x \in F \)?
   (c) If \( E \subseteq F \), is it necessarily true that \( E \in F \)?
   (d) If \( E \subseteq F \), is it necessarily true that \( E \subseteq F \)?