1 Exercise 1: Complex Deterministic FSA

The goal of this exercise is to show that the language L described below is a regular language by using the proof by construction in your Deterministic FSA notes. Assume $\Sigma = \{0, 1\}$.

$L = \{ w : w \text{ contains at least one 1 and } |w| \text{ starts with 0 or ends in 1} \}$

To do this, follow these steps:

i. Break the language L described by the complex property above into three languages $A$, $B$, $C$ described by simpler properties, so that $L = A \cap (B \cup C)$.

ii. Give the formal definitions and diagrams of the corresponding deterministic FSAs $M_A$, $M_B$, and $M_C$.

iii. Combine $M_B$ and $M_C$ into $M_{B\cup C}$ using the proof in your notes, specifying the formal description and diagram for $M_{B\cup C}$; then simplify this diagram and the description, if possible.

iv. Combine $M_A$ and $M_{B\cup C}$ into $M_{A\cap(B\cup C)}$ using the procedure in your notes, specifying the formal description and diagram for $M_{A\cap(B\cup C)}$; then simplify this diagram and the description, if possible.

2 Exercise 2: Complex Deterministic FSA.

Take language J, with alphabet $\Sigma = \{0, 1\}$. Interpret the English disjunction either...or in the defining property as exclusive or (i.e., interpret “either p or q” as meaning “p or q but not both”):

$J = \{ w : \text{either the length of } w \text{ is an even number or } w \text{ does not contain the sub-string 110} \}$

Break language J into two languages D and E described by two simpler properties, and give the formal definition and diagram for the corresponding deterministic FSA $M_D$ and $M_E$. Then, inspired by the proofs for the union and intersection of regular languages in your lecture notes, combine $M_D$ and $M_E$ into $M_{D\cup E}$ (where “?” is used as the set-theoretical operation corresponding to exclusive or). Provide the formal description and diagram for $M_{D\cup E}$. Simplify, if possible.
3 Exercise 3: Deterministic vs Non-Deterministic FSA.

For each of the following languages, construct a deterministic FSA that is most economical and a non-deterministic FSA with the indicated characteristics. Assume $\Sigma = \{a, b\}$. (You don’t need to use the proof for the operations $\cup$ and $\cap$ here.)

i. $\{w : w$ does not contain the sub-string $ba\}$. Non-det FSA with two states.

ii. The language $a(bb)^*$.  
   E.g. the strings $a, abb, aabbb, aabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbbbaabbb