1. Introduction

- Interpretation function: $[[\alpha]]^w$, $[[\alpha]]$
  $W$ is the set of logically possible worlds, $U$ is the universe of individuals, and $\{1, 0\}$ is the set of truth values.

(1) $[[\alpha]]^w = \text{xxx}$ is read as “the expression $\alpha$ denotes / stands for / refers to xxx in world $w$”.

(2) a. $[[\text{the president of the U.S. in Nov 2004}]]^{w_0} = \text{gwb}$
   b. $[[\text{the president of the U.S. in Nov 2004}]]^{w_1} = \text{jk}$

(3) a. $[[\text{the president of the U.S. in Nov 2004 is from Texas}]]^{w_0} = \text{TRUE} = 1$
   b. $[[\text{the president of the U.S. in Nov 2004 is from Texas}]]^{w_1} = \text{FALSE} = 0$

(4) $[[\alpha]] = \text{xxx}$ is read as “the expression $\alpha$ expresses xxx”.

(5) $[[\text{the president of the U.S. in Nov 2004}]]$
   = the function $f: W \rightarrow U$ such that, for every possible world $w \in W$,
     $f(w) = [[\text{the president of the U.S. in Nov 2004}]]^w$
     = whoever the president of the U.S. is in Nov 2004 in $w$

(6) $[[\text{the president of the U.S. in Nov 2004 is from Texas}]]$
   = the function $f: W \rightarrow \{1, 0\}$ such that, for every possible world $w \in W$,
     $f(w) = [[\text{the president of the U.S. in Nov 2004 is from Texas}]]^w$
     = the set $\{w: [[\text{the president of the U.S. in Nov 2004 is from Texas}]]^w = 1\}$

- The characteristic function of a set:

(7) a. Let $A$ be a set. The, char$_A$, the characteristic function of $A$, is the function $f$ such that,
    for any $x \in A$, $f(x) = 1$, and
    for any $x \notin A$, $f(x) = 0$.
   b. Let $f: A \rightarrow \{0,1\}$. Then, char$_f$, the set characterized by $f$, is
      $\{x \in A \mid f(x) = 1\}$
2. The Lexicon: the semantics of names, predicates and connectives.

- Proper names:
  At a given world \( w \), a proper name denotes / stands for / refers to a particular individual.

\[
[[\text{Jonathan D. Wright}]]^w = \varnothing \text{(you-know-who)}
\]

- Predicates: adjectives and verbs (and prepositions).
  A predicate denotes a set:
  - a set of individuals (for intransitive verbs or, more generally, 1-place predicates)

\[
\begin{align*}
\text{a. } [\text{female}]^w &= \{x : x \text{ is female in } w\} \\
\text{b. } [\text{female}]^{w_{23}} &= \{a, b\}
\end{align*}
\]

\[
\begin{align*}
\text{a. } [\text{run}]^w &= \{x : x \text{ runs in } w\} \\
\text{b. } [\text{run}]^{w_{23}} &= \{a, c\}
\end{align*}
\]

\[
\text{Universe in } w_{23}: \{a, b, c\}
\]

  - a set of pairs (for transitive verbs or 2-place predicates)

\[
\begin{align*}
\text{a. } [\text{fond-of}]^w &= \{<x,y> : x \text{ is fond of } y \text{ in } w\} \\
\text{b. } [\text{fond-of}]^{w_{23}} &= \{<a,b>, <b,c>, <c,c>, <c,a>\} \\
\text{Convention order: } SU \text{ Complement}
\end{align*}
\]

\[
\begin{align*}
\text{a. } [\text{love}]^w &= \{<x,y> : x \text{ loves } y \text{ in } w\} \\
\text{b. } [\text{love}]^{w_{23}} &= \{<a,b>, <b,c>, <c,c>\} \\
\text{Convention order: } SU \text{ DO}
\end{align*}
\]

  - a set of triples (for ditransitive verbs or 3-place predicates)

\[
\begin{align*}
\text{a. } [\text{send}]^w &= \{<x,y,z> : x \text{ sends } y \text{ to } z \text{ in } w\} \\
\text{b. } [\text{send}]^{w_{23}} &= \{<a,b,b>, <b,a,b>, <a,a,c>, <c,a,c>\} \\
\text{Convention order: } SU \text{ DO IO}
\end{align*}
\]

  - Etc.
QUESTION: Give the denotation of in-front-of.

(15)  a. \[[\text{in-front-of}]\]_w = 

     b. \[[\text{in-front-of}]\]_{w23} = 

Connectives: \text{and}_{\text{Boolean}}, \text{or}, \text{not}.

Connectives correspond to set-theoretical operations over sets. Take \text{X1}, \text{X2} and \text{X} below to be any expressions denoting sets.

(16)  a. \[[\text{X1 and X2}]\]_w = \[[\text{X1}]\]_w \cap \[[\text{X2}]\]_w

     b. \[[\text{X1 or X2}]\]_w = \[[\text{X1}]\]_w \cup \[[\text{X2}]\]_w

     c. \[[\text{not X}]\]_w = (\[[\text{X}]\]_w)’, i.e. the complement of \[[\text{X}]\]_w

(17)  Example:

       (Lucia is) Spanish or Italian

(18)  

\[ \begin{array}{c}
\text{S} \\
\text{q} \quad \text{p} \\
\text{NP} \quad \text{VP} \\
\text{g} \quad \text{w} \quad \text{q} \\
\text{N} \quad \text{AdjP} \\
\text{g} \quad \text{w} \quad \text{q} \\
\text{Lucia} \quad \text{AdjP} \quad \text{or} \quad \text{AdjP} \\
\text{g} \quad \text{g} \\
\text{Adj} \quad \text{Adj} \\
\text{g} \quad \text{g} \\
\text{Spanish} \quad \text{Italian} \\
\end{array} \]

QUESTION: The complex predicate in (19) is ambiguous. Give the two syntactic structures and compute their semantic denotation for an arbitrary \text{w} and for \text{w24}. Assume (20).

(19)  (Elena is) not tall and nice.

(20)  Universe in \text{w24}: \{a, b, c, d, e, f\}

      a, b and nobody else are tall in \text{w24}.
      b, c, d, f and nobody else are nice in \text{w24}.

Caveat! Not all occurrences of \text{and} correspond to \text{\cap}. Non-Boolean \text{and}, as in \text{John and Mary}, corresponds to the mereological sum operation \text{+}.
3. Semantic rules

- Semantic rules for combining predicates with their arguments:

\[(21) \quad \text{IP}^w \quad = \quad [[\text{NPsu}]]^w \in [[\text{VP}_1]]^w \]

\[ \text{NPsu} \quad \text{VP}_1 \]

\[(22) \quad \text{VP}^w \quad = \quad \{ x: <x, [[\text{NPod}]]^w> \in [[\text{V}_2]]^w \} \]

\[ \text{V}_2 \quad \text{NPod} \]

\[(23) \quad \text{VP}^w \quad = \quad \ldots \]

\[ \text{V}_3 \quad \text{NPod} \]

3.1. Example with a 1-place predicate.

\[(24) \quad \text{S} \]

\[ \text{NP} \quad \text{VP} \]

\[ \text{Ann} \quad \text{runs} \]

- Example with a 1-place predicate for world \(w_{23}\):

\[ [[\text{run}]]^{w_{23}} = \{a, c\} \]

\[ [[\text{Ann}]]^{w_{23}} = a \]

\[ [[\text{Ann runs}]]^{w_{23}} = [[\text{NP}]]^{w_{23}} \in [[\text{VP}]]^{w_{23}} \]

\[ = a \in [[\text{VP}]]^{w_{23}} \]

\[ = a \in \{a, c\} \]

\[ = \text{TRUE} = 1 \]

- Example with a 1-place predicate for an arbitrary world \(w\):

\[ [[\text{run}]]^w = \{x: x \text{ runs in } w\} \]

\[ [[\text{Ann}]]^w = a \]

\[ [[\text{Ann runs}]]^w = [[\text{NP}]]^w \in [[\text{VP}]]^w \]

\[ = a \in [[\text{VP}]]^w \]

\[ = a \in \{x: x \text{ runs in } w\} \]

\[ = a \text{ runs in } w \]
3.2. Example with a 2-place predicate.

\[(25)\]

\[
\begin{array}{ccc}
\text{w} & \text{S} \\
\text{NP} & \text{VP} \\
\text{g} & \text{e} & \text{i} \\
\text{Betty} & \text{V} & \text{NP} \\
g & g & g \\
\text{loves} & \text{Connor} \\
\end{array}
\]

- For world w\textsubscript{23}:

\[
[[\text{loves}]]\textsuperscript{w23} = \{<a,b>, <b,c>, <c,c>\}
\]

\[
[[\text{Connor}]]\textsuperscript{w23} = c
\]

\[
[[\text{loves Connor}]]\textsuperscript{w23} = \{ x: <x, [[\text{Connor}]]\textsuperscript{w23}> \in [[\text{love}]]\textsuperscript{w23} \}
\]

\[
= \{ x: <x, c> \in [[\text{love}]]\textsuperscript{w23} \}
\]

\[
= \{ x: <x, c> \in \{<a,b>, <b,c>, <c,c>\} \}
\]

\[
= \{ b, c \}
\]

\[
[[\text{Betty}]]\textsuperscript{w23} = b
\]

\[
[[\text{Betty loves Connor}]]\textsuperscript{w23} = [[\text{NP}]]\textsuperscript{w23} \in [[\text{VP}]]\textsuperscript{w23}
\]

\[
= b \in [[\text{VP}]]\textsuperscript{w23}
\]

\[
= b \in \{b, c\}
\]

\[
= \text{TRUE} = 1
\]

- For an arbitrary world w:

\[
[[\text{loves}]]\textsuperscript{w} = \{ <v, y>: v \text{ loves } y \text{ in } w \}
\]

\[
[[\text{Connor}]]\textsuperscript{w} = c
\]

\[
[[\text{loves Connor}]]\textsuperscript{w} = \{ x: <x, [[\text{Connor}]]\textsuperscript{w}> \in [[\text{love}]]\textsuperscript{w} \}
\]

\[
= \{ x: <x, c> \in [[\text{love}]]\textsuperscript{w} \}
\]

\[
= \{ x: <x, c> \in \{<v, y>: v \text{ loves } y \text{ in } w\} \}
\]

\[
= \{ x: x \text{ loves c in } w \}
\]

\[
[[\text{Betty}]]\textsuperscript{w} = b
\]

\[
[[\text{Betty loves Connor}]]\textsuperscript{w} = [[\text{NP}]]\textsuperscript{w} \in [[\text{VP}]]\textsuperscript{w}
\]

\[
= b \in [[\text{VP}]]\textsuperscript{w}
\]

\[
= b \in \{ x: x \text{ loves c in } w \}
\]

\[
= b \text{ loves c in } w
\]
QUESTION FOR RECITATION: Consider the complex predicate N’ old men and women from Barcelona and the two readings indicated in (26). Recall the syntactic trees for those two readings and spell out their semantic computation for w24 (assume the facts in (27) and for an arbitrary world w. To do the semantics, you will have to define in (28) a new semantic rule for combining N’ with modifiers (adjectives or PPs). Also, for the sake of this exercise, assume that the non-Boolean and in this example amounts to $\cup$.

(26) old men and women from Barcelona
   i. Reading (i): the men are old, the women are from Barcelona
   ii. Reading (ii): both the men and the women are old and from Barcelona

(27) World w24:
   Universe: \{a, b, c, d, e, f\}
   c, d and f are men.
   a, b and e are women
   c, d and a are old.
   c, d, f, a and e are from Barcelona.

(28) N’ modification rule:

\[ N' \quad \ W = \quad q \quad \ w \quad p \quad \ \text{Mod}_1 \quad \ldots \quad N' \quad \ldots \quad \text{Mod}_n \]