Introduction to Predicate Logic  
Ling 106  
November 12, 2003

1. Basic characteristic of Predicate Logic (PrL).  
Besides keeping the connectives from PL, Predicate Logic decomposes simple statements into smaller parts: predicates, terms, and quantifiers.

(1) John is tall. \( T(j) \)
(2) John is taller than Bill \( T(j, b) \)
(3) Everybody sleeps \( \forall x[S(x)] \)
(4) Somebody likes David. \( \exists x[L(x, d)] \)

2. Syntax of PrL.

Primitive vocabulary:
Lexical entries, with a denotation of their own:
- a set of individual constants, represented with the letters a, b, c, d…
- a set of individual variables \( x_0, x_1, x_2, \ldots y_0, y_1, y_2 \ldots \) Individual constants and individual variables together constitute the set of terms.
- A set of predicates, each with a fixed n-arity, represented by \( P, Q, R, \ldots \)

Symbols: The PL logical connectives
The quantifier symbols \( \exists \) and \( \forall \).

Syntactic rules:
If \( P \) is a n-ary predicate and \( t_1 \ldots t_n \) are all terms, then \( P(t_1 \ldots t_n) \) is an atomic formula.
If \( p \) is a formula, then \( \neg p \) is a formula.
If \( p \) and \( q \) are formulae, then \( (p \land q), (p \lor q), (p \rightarrow q), (p \leftrightarrow q) \) are formulae too.
If \( p \) is a formula and \( x \) is a variable, then the following expressions are formulae:
\[ \forall x \; p, \; \exists x \; p. \]
Nothing else is a formula in PrL.

Question:
Translate into PrL the following English sentences:
- a. John likes Susan.
- b. John has a cat that he spoils.
- c. Everything is bitter or sweet.
- d. Either everything is bitter or everything is sweet.
- e. There is something that everybody told Mary.
- f. Everybody told Mary something.
- g. If all logicians are smart, then Alfred is smart too.
- h. Nobody came.
- i. Nobody is loved by no one.
1. Meanings of denotations of lexical items.

What are meanings? What do linguistic expressions “stand for” or “denote”?

Some small phrases and words can be used to stand for or denote a concrete individual (or for a group of them) in the world. Instead of using that word or phrase, you could simply point at the real object in the actual world. The following are some examples:

(1) Proper names: Philadelphia, Morocco, Delaware River, Williams Hall
(2) Noun Phrases with the definite article: the highest mountain in Pennsylvania, the tallest spy, the president of Italy, the students of Ling 106.
(3) Noun phrases with demonstratives: This table here, that window over there, these chairs, those pens
(4) Pronouns: I, you, he, us, them

Some other words and phrases, though, do not stand or denote a concrete object:

(5) Non-referential noun phrases:
   Nothing is trivial
   No student is sick
   Every woman talked to the cat sitting on her lap
(6) Verbs, nouns, adjectives, prepositions:
   Run, see, put, student, female, red, tall, with, in front of.

Current semantic theory proposes to treat meanings as set-theoretic objects. Some noun phrases stand for or denote concrete individuals in the world, but other phrases denote more abstract entities: e.g. a set of individuals, or a relation between two sets of individuals.

||student|| = {John, Scott, Lisa, Nichola}
||female|| = {Lisa, Nichola}
||see|| = {<John, Nichola>, <Lisa, John>, <John, Lisa>, <Nichola, John>, …}