1 Exercise 1

Given the sets $A$, $B$, $C$ and $D$, classify the statements below as either true or false.

$A = \{1, 2, 3\}$

$B = \{1, 3, 4, 5, \{2\}, \{1, 2\}, \emptyset\}$

$C = \{1, 3\}$

$D = \{1, 2, \{\emptyset\}\}$

Statements:

- a) $1 \in A$
- b) $3 \in A$
- c) $3 \subseteq A$
- d) $\{3\} \subseteq A$
- e) $\{3\} \in A$
- f) $A \subseteq B$
- g) $A \subseteq C$
- h) $C \subseteq B$
- i) $C \subseteq A$
- j) $D \subseteq A$
- k) $2 \in B$
- l) $\{2\} \in B$
- m) $\{1, 2\} \in B$
- n) $\{2\} \subseteq B$
- o) $\{1, 2\} \subseteq B$
- p) $\emptyset \subseteq B$
- q) $\emptyset \in D$
- r) $\emptyset \subseteq D$
- s) $\{\emptyset\} \in D$
- t) $\{\emptyset\} \subseteq D$

2 Exercise 2

Take the sets $H$, $I$, $J$ and $K$. The universe is $\{a, \{b\}, c, d, e, \{b, c\}\}$. Specify the sets below:

$H = \{a, c\}$

$I = \{a, \{b\}, c, d, e\}$

$J = \{a, \{b, c\}\}$

$K = \{a, \{b\}\}$

Sets:

- a) $H \cap I$
- b) $H \cup J$
- c) $H - J$
- d) $I \cup H$
- e) $I - H$
- f) $I - K$
- g) $J - K$
- h) $J \cup K$
- i) $I^c \cup K$
- j) $\varnothing(J)$
3 Exercise 3

For any arbitrary sets $A$, $B$ and $C$, simplify the following expression as much as possible by using exclusively the set-theoretical equalities in p. 18 of the reading. (If you want to use something else, you will have to prove it first.) Apply only one type of equality per line.

$$(A - (B - C)) \cap ((A \cap B) \cup C)$$

4 Exercise 4

Show that, for any sets $A$, $B$ and $C$, the following statement is true. Use exclusively the set-theoretical equalities in p. 18 of the reading. (If you want to use something else, you will have to prove it first.) Apply only one type of equality per line.

$$((A \cap B) - C) \subseteq ((A \cup B) - (A \cap C))$$

5 Exercise 5

Given the sets $Q$ and $R$, compute the Cartesian product $Q \times R$. Then, take the relations $T \subseteq Q \times R$ and $S \subseteq Q \times R$ and specify the inverse of $T$, $T^{-1}$, and the complement of $S$, $S'$. For each of these four relations, determine (i) whether it is a function or not, and, if it is, (ii) whether it is "into" or "onto" and (iii) whether it is many-to-one or one-to-one.

- $Q = \{a, b, c, d\}$  
  $R = \{1, 2, 3\}$  
  $Q \times R =$

- $T = \{< a, 1 >, < b, 3 >, < c, 2 >, < d, 2 >\}$  
  $T^{-1} =$

- $S = \{< a, 1 >, < a, 2 >, < a, 3 >, < b, 1 >, < b, 2 >, < c, 1 >, < c, 3 >, < d, 2 >, < d, 3 >\}$  
  $S' =$