Some in-class exercises
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1 Exercise 1

Given the sets \( A, B, C, D \) and \( E \), classify the statements below as either true or false.
\[
A = \{a, b\}
\]
\[
B = \{a, b, c, d\}
\]
\[
C = \{a, b, \{b\}\}
\]
\[
D = \{a, b, \emptyset\}
\]
\[
E = \{b\}
\]

Statements:

a) \( d \in A \) false  

b) \( c \in A \) false  

c) \( c \in B \) true  

d) \( \{b\} \in E \) false  

e) \( \{a, d\} \in A \) false  

f) \( b \subseteq E \) false  

g) \( \{a, b, c, d\} \subseteq B \) true  

h) \( \{a, b\} \in B \) false  

i) \( \{a, \emptyset\} \subseteq D \) true  

j) \( \{a, c\} \subseteq C \) false  

k) \( \emptyset \in A \) false  

l) \( \emptyset \subseteq B \) true  

m) \( \{\emptyset\} \subseteq D \) true  

n) \( \{\emptyset\} \subseteq C \) false  

o) \( A \subseteq B \) true  

p) \( A \in B \) false  

q) \( D \subseteq C \) false  

r) \( E \in A \) false  

s) \( E \subseteq C \) true  

t) \( E \in C \) true

2 Exercise 2

Take the sets \( F, G, H \) and \( I \) and assume that the universe of discourse is \( \bigcup \{F, G, H, I\} \). Specify the sets below:
\[
F = \{1, 2, 3, 4, 5\}
\]
\[
G = \{1, 2, \{1\}, \{1, 3\}\}
\]
\[
H = \{3, 4, 5\}
\]
\[
I = \{1, 3\}
\]

Sets:

a) \( F \cup G = \{1, 2, 3, 4, 5, \{1\}, \{1,3\}\} \)

b) \( F - G = \{3, 4, 5\} \)

c) \( G - F = \{\{1\}, \{1,3\}\} \)

d) \( H - G = H = \{3, 4, 5\} \)

e) \( F' = \{\{1\}, \{1,3\}\} \)

f) \( (G \cup I) \cap H = \{3\} \)

g) \( H \cap G = \emptyset \)

h) \( I' \cap H = \{4, 5\} \)

i) \( G \cap I' = \{2, \{1\}, \{1,3\}\} \)

j) \( (\phi(H)) \cap G = \emptyset \)
3 Exercise 3

The symmetric difference of any two sets $A$ and $B$, denoted $A+B$, is defined as the set whose members are in $A$ or in $B$ but not in both $A$ and $B$, i.e.:

$$A + B = \text{def} \ (A \cup B) - (A \cap B)$$

Show that, for any sets $A$ and $B$, the following statement is true. Use exclusively the set-theoretical equalities in p. 18 of the reading and the definition above (and the definition of difference!). (If you want to use something else, you will have to prove it first.)

$$A + B = (A - B) \cup (B - A)$$

1. $A + B$
2. $(A \cup B) - (A \cap B)$ \hspace{2cm} \text{Def +}
3. $(A \cup B) \cap (A \cap B)'$ \hspace{2cm} \text{Def -}
4. $(A \cup B) \cap (A' \cup B')$ \hspace{2cm} \text{DeMorgan}
5. $[(A \cup B) \cap A'] \cup [(A \cup B) \cap B']$ \hspace{2cm} \text{Distr}
6. $[A' \cap (A \cup B)] \cup [B' \cap (A \cup B)]$ \hspace{2cm} \text{Comm 2x}
7. $[(A' \cap A) \cup (A' \cap B)] \cup [B' \cap (A \cup B)]$ \hspace{2cm} \text{Distr}
8. $[\emptyset \cup (A' \cap B)] \cup [B' \cap (A \cup B)]$ \hspace{2cm} \text{Compl}
9. $(A' \cap B) \cup [B' \cap (A \cup B)]$ \hspace{2cm} \text{Ident}
10. $(A' \cap B) \cup [(B' \cap A) \cup (B' \cap B)]$ \hspace{2cm} \text{Distr}
11. $(A' \cap B) \cup [(B' \cap A) \cup \emptyset]$ \hspace{2cm} \text{Compl}
12. $(A' \cap B) \cup (B' \cap A)$ \hspace{2cm} \text{Ident}
13. $(A \cap B') \cup (B \cap A')$ \hspace{2cm} \text{Comm 3x}
14. $(A - B) \cup (B - A)$ \hspace{2cm} \text{Def -}

4 Exercise 4

Apply Harris conditions I and II to show whether the word *hopeful* consists of one single morpheme or it consists of two, namely *hope* + *ful*. Be as explicit as the lectures notes. In particular:

(i) specify your environment $\underline{\text{X}}$;

(ii) say what $A$, $B$, $C$ and $D$ are (provide a suitable $C$ and $D$ yourself);

(iii) give the “crossed” examples that to prove condition I (give both “crossings”!);

(iv) build your sets $\alpha$ and $\beta$ and spell out several environments where you test the categorial behavior required in condition II;

(v) enunciate the conclusion derived by the distributional method.