1. Basic characteristic of Predicate Logic (PrL).

- Besides keeping the connectives from PL, Predicate Logic (PrL) decomposes simple statements into smaller parts: predicates, terms and quantifiers.

(0) John is tall.
    T(j)

(1) John is taller than Bill.
    T(j,b)

(2) Everybody sleeps.
    \forall x \ [S(x)]

(3) Somebody likes David.
    \exists x \ [L(x,d)]

2. Syntax of PrL.

- Primitive vocabulary:

(4) Lexical entries, with a denotation of their own:
   a. A set of individual constants, represented with the letters a, b, c, d…
   b. A set of individual variables x_0, x_1, x_2, … y_0, y_1, y_2, … Individual constants and individual variables together constitute the set of terms.
   c. A set of predicates, each with a fixed n-arity, represented by P, Q, R …

(5) Symbols treated syncategorematically:
   a. The PL logical connectives.
   b. The quantifier symbols \exists and \forall.

- Syntactic rules:

(6) a. If P is a n-ary predicate and t_1… t_n are all terms, then P(t_1… t_n) is an atomic formula.
   b. If \phi is a formula, then \neg \phi is a formula.
   c. If \phi and \psi are formulae, then (\phi \land \psi) are formulae too.
       (\phi \lor \psi)
       (\phi \rightarrow \psi)
       (\phi \leftrightarrow \psi)
   d. If \phi is a formula and v is a variable, then the following expressions are formulae:
       \forall v \phi  “For every v, it holds that \phi”.
       \exists v \phi  “There is some v for which it holds that \phi”.
   e. Nothing else is a formula in PrL.
QUESTION: Draw the syntactic tree for the expressions in (8) that are well-formed formulae of PrL.

(8)

a. \( \exists \forall (Qa \rightarrow PR(b)(c)) \)

b. \( \forall x (P(x) \rightarrow \exists y Q(x,y)) \)

c. \( \exists x_1 \forall x_2 (P(x_1,x_2) \rightarrow (R(x_1) \land Q(x_2,a))) \)

QUESTION: Translate into PrL the following English sentences:

(9)

a. John likes Susan.

b. John has a cat that he spoils.

c. Everything is bitter or sweet.

d. Either everything is bitter or everything is sweet.

e. There is something that everybody told Mary.

f. Everybody told Mary something.

g. If all logicians are smart, then Alfred is smart too.

h. Nobody came.

i. Nobody is loved by no one.

j. John visited Mary in Barcelona.

k. John saw the man with the binoculars.

l. John saw the pig from the barn on the hill.

Some syntactic notions:

(10) If \( x \) is any variable and \( \varphi \) is a formula to which a quantifier has been attached by rule (7.d) to produce \( \forall x \varphi \) or \( \exists x \varphi \), then we say that \( \varphi \) is the scope of the attached quantifier and that \( \varphi \) or any part of \( \varphi \) lies in the scope of that quantifier.

(11) An occurrence of a variable \( x \) is bound if it occurs in the scope of \( \forall x \) or \( \exists x \). A variable is free if it is not bound.