1. Meanings or denotations of lexical items.

- What are meanings? What do linguistic expressions “stand for” or “denote”? 

- Some small phrases and words can be used to stand for or denote a concrete individual (or for a group of them) in the world. Instead of using that word or phrase, you could simply point at the real object in the actual world. The following are some examples:

  (1) Proper names:
      Maribel, Philadelphia, Morocco, Delaware River, Williams Hall.

  (2) Noun Phrases with the definite article (=definite descriptions):
      The highest mountain in Pennsylvania, the tallest spy, the president of Italy, the students of Ling 300.

  (3) Noun phrases with demonstratives:
      This table here, that window over there, these chairs, those pens.

  (4) Pronouns:
      I, you, he, us, them…

- Some other words and phrases, though, do not stand or denote a concrete object:

  (5) Non-referential Noun Phrases:
      a. Nothing is trivial.
      b. No student is sick.
      c. Every woman, talked to the cat sitting on her lap.

  (6) Verbs, nouns, adjectives, prepositions:
      Run, see, put, student, female, red, tall, with, in front of.

- Current semantic theory proposes to treat meanings as set-theoretic objects. Some Noun Phrases stand for or denote concrete individuals in the world, but other phrases denote more abstract entities: e.g., a set of individuals, or a relation between two sets of individuals.

  (7) \([\text{student}]\) = \{Ben, Dave, Katherine, Kathy, Naomi, Solaman, Brian, Jana, Shaleigh, Lauren\}

  (8) \([\text{female}]\) = \{Jana, Shaleigh, Lauren, Sue, Kathy, Naomi\}
2. Sets, characteristic functions and schönfinkelization.

■ Characteristic function of a set.  
Every set $A$ can be transformed into a function –called characteristic function of $A$-- whose range is $\{0,1\}$, in the following way:

(11) Let $A$ be a set. The, $\text{char}_A$, the characteristic function of $A$, is the function $f$ such that, for any $x \in A$, $f(x)=1$, and for any $x \notin A$, $f(x)=0$.

(12) The set denoted by $\textbf{female}$:  
$A_{\text{female}} = \{\text{Jana, Shaleigh, Lauren, Sue, Kathy, Naomi}\}$

(13) $U = \{\text{Ben, Dave, Jana, Shaleigh, Brian, Lauren, Sue, Kathy, Naomi}\}$

(14) The characteristic function of $A_{\text{female}}$:

\[
\begin{align*}
\text{Ben} & \rightarrow 0 \\
\text{Dave} & \rightarrow 0 \\
\text{Jana} & \rightarrow 1 \\
\text{Shaleigh} & \rightarrow 1 \\
\text{Brian} & \rightarrow 0 \\
\text{Lauren} & \rightarrow 1 \\
\text{Sue} & \rightarrow 1 \\
\text{Kathy} & \rightarrow 1 \\
\text{Naomi} & \rightarrow 1 \\
\end{align*}
\]

■ Schönfinkelization.  
Any $n$-ary function can be turned into a multiple embedding $1$-place function.

(15) $U = \{a,b,c\}$
(16) The relation denoted by see:
\[ R_{\text{see}} = \{ <a,b>, <b,c>, <c,c> \} \]

(17) The characteristic function of \( R_{\text{see}} \):
\[
\begin{align*}
<a,a> & \rightarrow 0 \\
<a,b> & \rightarrow 1 \\
<a,c> & \rightarrow 0 \\
<b,a> & \rightarrow 0 \\
<b,b> & \rightarrow 0 \\
<b,c> & \rightarrow 1 \\
<c,a> & \rightarrow 0 \\
<c,b> & \rightarrow 0 \\
<c,c> & \rightarrow 1 
\end{align*}
\]

(18) Turning n-ary functions into multiple embedded 1-ary functions: Schönfinkelization.

a. Left-to-right:
\[
\begin{align*}
a & \rightarrow 0 \\
a & \rightarrow b \rightarrow 1 \\
c & \rightarrow 0 \\
\end{align*}
\]

b. Right-to-left: (inverse + left-to-right shonf.)
\[
\begin{align*}
a & \rightarrow 0 \\
a & \rightarrow b \rightarrow 0 \\
c & \rightarrow 0 \\
\end{align*}
\]

(19) \( U = \{ d, e \} \)

(20) The relation denoted by show:
\[
R_{\text{show}} = \{ <d,d,d>, <d,e,d>, <e,d,d>, <e,e,e>, <e,e,d> \}
\]
\[ <x,y,z> \in R_{\text{show}} \text{ iff } x \text{ shows } y \text{ to } z \]
3. Lambda notation for functions.

Sometimes we know intuitively what the set/function denoted by a given linguistic expression should look like, even though we may not know its full spelled out form or it would be too long to list.

In this case, we can use property notation for sets and (lambda) \( \lambda \)-notation for functions.

- Property notation for sets.
  
  (21)  
  a. \{3, 6, 9, 12, 15, 18, \ldots \}  
  b. \{x: \text{x is a natural number multiple of 3}\}

(22)  
 a. [[female]] = \{Jana, Shaleigh, Lauren, Sue, Kathy, Naomi\}  
 b. [[female]] = \{x: \text{female(x)}\}

- (Lambda) \( \lambda \)-notation for functions.

(21) \( \lambda x. \phi = \) the function that, for every individual \( v \) in \( U \), maps \( v \) into \( \phi' \), where \( \phi' \) is the result of writing \( v \) instead of \( x \) in the original \( \phi \).

(22) [[female]] = \( \lambda x. \text{female(x)} \)

(23) [[see]] = \( \lambda x. \lambda y. \text{see(y,x)} \) (for the R_{see} schonfinkelized right-to-left)

4. Building the meaning of sentences compositionally.

- Example 1:

(24) Brian is a student.

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        IP
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Example 2:

(25) Kathy is a female student

\[
\text{Kathy} \in \{x: \text{female}(x)\} \land \{x: \text{student}(x)\}
\]
Example 3:

(26) Maribel sees Andreas.

\[
\begin{align*}
\text{S} & \quad \text{??}
\end{align*}
\]

\[
\begin{align*}
\text{NP}_{\text{su}} & \quad \text{VP} & \quad \text{??}
\end{align*}
\]

\[
\begin{align*}
\text{Maribel} & \quad \text{V} & \quad \text{NP}
\end{align*}
\]

\[
\begin{align*}
\text{sees} & \quad \text{Andreas}
\end{align*}
\]

\[
\begin{align*}
m & \quad \{<y,x>: \text{see}(y,x)\} & \quad a
\end{align*}
\]

\[
\begin{align*}
\text{S} & \quad \lambda y.\text{see}(y,a) \ (m) \Rightarrow \text{see}(m,a)
\end{align*}
\]

\[
\begin{align*}
\text{NP}_{\text{su}} & \quad \text{VP} & \quad \lambda x.\lambda y.\text{see}(y,x) \ (a) \Rightarrow \lambda y.\text{see}(y,a)
\end{align*}
\]

\[
\begin{align*}
\text{Maribel} & \quad \text{V} & \quad \text{NP}
\end{align*}
\]

\[
\begin{align*}
\text{sees} & \quad \text{Andreas}
\end{align*}
\]

\[
\begin{align*}
m & \quad \lambda x.\lambda y.\text{see}(y,x) & \quad a
\end{align*}
\]

Example 4:

(27) Maribel saw the man.

\[
\begin{align*}
\text{S} & \quad \text{see}(m,x,e) \land \text{man}(x) \land \text{PAST}(e)
\end{align*}
\]

\[
\begin{align*}
\text{NP} & \quad \text{TenseP} & \quad \lambda y. \text{see}(y',x,e) \land \text{man}(x) \land \text{PAST}(e)
\end{align*}
\]

\[
\begin{align*}
m & \quad \text{Maribel} & \quad \text{T} & \quad \text{VP} & \quad \lambda e'.\lambda y'. \text{see}(y',x,e') \land \text{man}(x)
\end{align*}
\]

\[
\begin{align*}
\text{PAST} & \quad \text{see} & \quad \text{NP} & \quad x & \quad \text{man}(x)
\end{align*}
\]

\[
\begin{align*}
\text{the} & \quad \text{man}
\end{align*}
\]

\[
\begin{align*}
e & \quad \lambda x'.\lambda e'.\lambda y'. \text{see}(y',x',e') & \quad \lambda v. \text{man}(v)
\end{align*}
\]
Example 5:

(28) Sophia saw the man with the binoculars.  (Reading where the man has the binoculars)

\[
S \quad \text{see}(s,x,e) \land \text{man}(x) \land \text{has/with}(x,r) \land \text{bin}(r) \land \text{PAST}(e)
\]

NP  

TenseP  \(\lambda y'. \text{see}(y',x,e) \land \text{man}(x) \land \text{has/with}(x,r) \land \text{bin}(r) \land \text{PAST}(e)\)

| Sophia

T VP  \(\lambda e'. \lambda y'. \text{see}(y',x,e') \land \text{man}(x) \land \text{has/with}(x,r) \land \text{bin}(r)\)

PAST see NP x \(\text{man}(x) \land \text{has/with}(x,r) \land \text{bin}(r)\)

the N' \(\lambda x'. \text{man}(x') \land \text{has/with}(x',r) \land \text{bin}(r)\)

N PP \(\lambda x'. \text{has/with}(x',r) \land \text{bin}(r)\)

man with NP r \(\text{bin}(r)\)

\(\lambda x'. \text{man}(x')\)

the binoculars

\(\lambda r. \text{binoculars}(r)\)

\(\lambda x'. \lambda e'. \lambda y'. \text{see}(y',x',e')\)

\(\lambda z'. \lambda x'. \text{has/with}(x',z')\)
Example 6:

(29) Sophia saw the man with the binoculars.  (Reading where Sophia has the binoculars)

```
 S           see(s,x,e) ∧ man(x) ∧ has/with(e,r) ∧ bin(r) ∧ PAST(e)
 NP          TenseP λy'. see(y',x,e) ∧ man(x) ∧ has/with(e,r) ∧ bin(r) ∧ PAST(e)
 Sophia     T         VP      λe'.λy'. see(y',x,e') ∧ man(x) ∧ has/with(e',r) ∧ 
 s           PAST     V'       PP               bin(r)
 e           PAST(e)  V         NP               P         NP
 see       the       man     with    the    binoculars
 ↓             ↓          ↓            ↓
 λe'.λy'. see(y',x,e') ∧ man(x)   λe'.has/with(e',r) ∧ bin(r)
```