Homework Assignment 2:
Propositional Logic, Lexical Analysis
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Oct. 1, 2001
Due on Oct 8, 2001 by 1pm

1 Exercise 1: Syntax of PL

Establish which of the following expressions are well-formed formulae in Propositional Logic and which are not.

a) \( p \)

f) \( (p \land r) \)

b) \( (q \land p) \)

g) \( (p \land (r \rightarrow q)) \)

c) \( \land \)

h) \( (p \lor q') \)

d) \( ((s \rightarrow t) \lor \neg p) \)

i) \( (\neg t \leftrightarrow ((p \land q) \rightarrow s)) \)

e) \( \neg(p \land q \lor r) \)

j) \( ((p \leftrightarrow q) \land r)) \)

2 Exercise 2: Semantics of PL.

Construct a truth table for each of the following statements:

a) \( ((p \lor r) \rightarrow \neg p) \)

b) \( ((p \land q) \rightarrow (p \lor r)) \)

c) \( ((p \rightarrow (q \lor \neg r)) \land (p \rightarrow (q \lor \neg r))) \)

d) \( (((p \rightarrow q) \rightarrow p) \rightarrow q) \)

3 Exercise 3: Semantics of PL.

Using truth tables, show that the following formulae are logically equivalent:

\( (p \leftrightarrow q) \Leftrightarrow ((\neg p \land \neg q) \lor (p \land q)) \)

4 Exercise 4: Semantics of PL.

Define a truth table for the complex English expression except-if. E.g., in a sentence like Mary will go on vacation except if she is broke.

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \psi )</th>
<th>( \phi \text{ except-if } \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
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</table>
5 Exercise 5: Semantics of PL

A useful Boolean operation is Quine's dagger \( \downarrow \), whose nearest English correspondent is *neither... nor*:

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \psi )</th>
<th>( \phi \downarrow \psi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td>1</td>
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<td>1</td>
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</tbody>
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An interesting characteristic of this connective is that the five typical Boolean connectives can be defined by using just \( \downarrow \). Since we already saw in class that all other connectives can be defined in terms of \( \neg \) and \( \lor \), we only need to show that \( \neg \) and \( \lor \) can be defined in terms of \( \downarrow \). Do that by: (i) showing how \( \neg \phi \) can be expressed with Quine’s dagger, (ii) defining \( \phi \lor \psi \) using only \( \neg \) and \( \downarrow \), and (iii) defining \( \phi \lor \psi \) in terms of \( \downarrow \) alone.

6 Exercise 6: Grammaticality

Give an English-looking expression that exemplifies each of the following categories (obviously, the examples discussed in class do not count):

1. Descriptively ungrammatical expression under any dialect of English.

2. Descriptively grammatical sentence, and prescriptively ungrammatical.

3. Grammatical sentence with semantic anomaly.

4. Grammatical sentence with processing difficulty.

7 Exercise 7: Lexical Analysis

Do exercise 2.1-8 (Palaun) in Langacker’s chapter p. 52.

8 Exercise 8: Lexical Analysis

Do exercise 2.1-9 (Classical Aztec) in Langacker’s chapter p. 52-3.