Homework Assignment 1. Solutions
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1 Exercise 1

Given the sets $A$, $B$, $C$, $D$ and $E$, classify the statements below as either true or false.

$A = \{a, b, c\}$
$B = \{a, b, c, d\}$
$C = \{a, b, \{b\}\}$
$D = \{a, b, \emptyset\}$
$E = \{b\}$

Statements:

False  a) $d \in A$
True  b) $c \in A$
True  c) $c \in B$
False  d) $\{b\} \in E$
False  e) $\{a, d\} \in A$
False  f) $b \subseteq E$
True  g) $\{a, b, c, d\} \subseteq B$
False  h) $\{a, b\} \in B$
True  i) $\{a, \emptyset\} \subseteq D$
False  j) $\{a, c\} \subseteq C$
False  k) $\emptyset \in A$
True  l) $\emptyset \subseteq B$
True  m) $\{\emptyset\} \subseteq D$
False  n) $\{\emptyset\} \subseteq C$
True  o) $A \subseteq B$
False  p) $A \in B$
False  q) $D \subseteq C$
False  r) $E \in A$
True  s) $E \subseteq C$
True  t) $E \in C$

2 Exercise 2

Take the sets $F, G, H$ and $I$ and assume that the universe of discourse is $\bigcup\{F, G, H, I\}$. Specify the sets below:

$F = \{1, 2, 3, 4, 5\}$
$G = \{1, 2, \{1\}, \{1, 3\}\}$
$H = \{3, 4, 5\}$
$I = \{1, 3\}$

Universe = $\{1, 2, 3, 4, 5, \{1\}, \{1, 3\}\}$

Sets:
3 Exercise 3

Show that, for any sets $A, B$ and $C$, the following statement is true. Use exclusively the set-theoretical equalities in p. 18 of the reading. (If you want to use something else, you will have to prove it first.)

[(A \cap B) - C] \subseteq (B - C) \\
By consistency, we show: 
\[(A \cap B) - C\cap (B - C) = (A \cap B) - C\]
\[((A \cap B) - C) \cap (B - C)\]
\[((A \cap B) \cap C')] \cap (B \cap C')\] Diff
\[(A \cap (B \cap B)) \cap (C' \cap C')\] Comm, assoc several times
\[(A \cap B) \cap C'\] Idemp
\[(A \cap B) - C\] Diff

4 Exercise 4

For any arbitrary sets $A, B$ and $C$, simplify the following expression as much as possible by using exclusively the set-theoretical equalities in p. 18 of the reading.

\[(A - C) \cup (B - C)\]
\[(A \cap C') \cup (B \cap C')\] Diff
\[(C' \cap A) \cup (C' \cap B)\] Comm
\[C' \cap (A \cup B)\] Distrib
\[(A \cup B) \cap C'\] Comm
\[(A \cup B) - C\] Diff

5 Exercise 5

Given the sets $O$ and $P$, compute the Cartesian product $O \times P$. Then, take the relations $R : O \rightarrow P$ and $S : O \rightarrow P$ and specify the inverse of $R, R^{-1}$, and the complement of $S, S'$. For each of these four relations, determine (i) whether it is a function or not, and, if it is, (ii) whether it is “into” or “onto” and (iii) whether it is many-to-one or one-to-many.

- $O = \{a, b, c\}$
- $P = \{1, 2, 3\}$
- $O \times P = \{<a, 1>, <a, 2>, <a, 3>, <b, 1>, <b, 2>, <b, 3>, <c, 1>, <c, 2>, <c, 3>\}$

- $R = \{<a, 1>, <a, 3>, <b, 2>\}$ Not a function.
- $R^{-1} = \{<1, a>, <3, a>, <2, b>\}$ Function, into, many-to-one.
- $S = \{ <a, 2>, <a, 3>, <b, 1>, <b, 2>, <c, 1>, <c, 3> \}$ Not a function.
  $S' = \{ <a, 1>, <b, 3>, <c, 2> \}$ Function, onto, one-to-one (i.e. not one-to-many, and technically not many-to-one).