A Formalist Perspective on Language Acquisition

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Abstract

Language acquisition is a computational process by which linguistic experience is integrated into the learner's initial stage of knowledge. To understand language acquisition thus requires precise statements about these components and their interplay, stepping beyond the philosophical and methodological disputes such as the generative vs. usage-based approaches. I review several mathematical models that have guided the study of child language acquisition: How learners integrate experience with their prior knowledge of linguistic structures, How researchers assess the progress of language acquisition with rigor and clarity, and How children form the rules of language even in the face of exceptions. I also suggest that these models are applicable to second language acquisition (L2), yielding potentially important insights on the continuities and differences between child and adult language.

1 A Formal Introduction

Don Knuth, perhaps the most renowned living computer scientist, infamously took *Syntactic Structures* on his honeymoon. For many language scientists, and for many more outside of linguistics, generative grammar initiated an exciting way of studying a quintessential aspect of human life. It was a refreshing change from the routine in traditional social and behavioral sciences: take measurements, fit a curve, repeat. While the study of language has an ancient tradition, it is safe to say that it was the commitment to a causal and mechanical account of language helped establish the field of cognitive science; the mind may be studied with deductive methods as in the natural sciences.

This article provides a personal perspective on the formalist approach to language acquisition. I use the term "formalist" in the methodological sense: it reflects a commitment to the mechanistic study of language, rather than the conventionalized standin for "generative" and/or "nativist". In my opinion, methodological rigor should override the researcher's preference for the mode of explanation. Indeed, as someone trained in the generative tradition which emphasizes domain-specific knowledge, I have been invigorated by usage-based research which places

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greater emphasis on the role of data-driven learning. More specifically, a formal theory of language acquisition ought to be more than a description, that children know A at age X but B at age $X+Y$ (e.g., the transition from an item-based to a rule-based grammar), or that some variable P (e.g., input frequency) is correlated with some other variable Q (e.g., the correct rate of morphological marking). The theory should also include a mechanistic account of how the $A \rightarrow B$ transition takes place and how P causally affects Q. After all, it was such a mechanistic approach that got everyone excited about linguistics and helped build connections with other fields.

In the spirit of being formal, I have organized this paper around three simple mathematical models. As I will review in Section 2, 3, and 4, they have been developed to address the so-called logical problem of language acquisition (Hornstein and Lightfoot 1981), that the language learner must go beyond their linguistic experience to form general linguistic rules. In Section 5, I submit, with some trepidation, that these equations may also prove useful for L2 acquisition. This is because learning a second language must also go beyond the data to form generalizations – the logical problem of L2 acquisition (White 1985, Bley-Vroman 1989). The equations provide very concrete predictions that can be easily confirmed or disconfirmed. To the extent they are confirmed, we may detect potential continuities between child and adult language acquisition. To the extent they are disconfirmed, we may be a step closer to understanding why children seem better at language learning than adults.

2 Competition and Selection

2.1 Background

There was a time when formal methods were integral to the empirical research on child language: I have in mind the influential work of Suppes (1974), Pinker (1979), Wexler and Culicover (1980), Berwick (1985), among others. Formal models make explicit statements about the learner's predisposition for language, the ecological condition of language acquisition (e.g., no negative evidence), and the learning algorithms likely within children's computational capacity (e.g., incremental learning). The commitment to a formal account of language learnability in fact inspired the modern study of machine learning and statistical inference (Solomonoff 1964, Gold 1967, Blum and Blum 1975, Angluin 1980).

The scene had already changed when I started working on child language in the late 1990s. The field, at least the part of the field I was embedded in, had shifted primarily to developmental issues. A major debate at the time was whether the differences between child and adult language are due to competence or performance gaps (e.g., Pinker 1984, Borer and Wexler 1987, Demuth 1989, Bloom 1990, Valian 1991, Wang et al. 1992, Hyams and Wexler 1993). But both sides of the debate agreed that the child's grammar is already target-like as soon as it can be directly tested (e.g., at the beginning of multi-word combinations). This was a natural position for the performance-based theorists but seemed paradoxical under the competence-based account: if children have exquisite knowledge of their grammar (Wexler 1998), how come they don't talk right? Conspicuously absent is a learnability account of *how* children's grammar becomes target-like: both sides in effect deny the role of input and experience, because language-specific data has no explanatory power on the acquisition of the grammar.[1]

[1]Under some accounts, children's output is constrained by performance filters that are themselves subject to lan-

As someone on the outside looking in, this state of affairs was fascinating but also puzzling. The quantitative work that began to emerge, thanks in no small part to the CHILDES project (MacWhinney 2000), confirmed that many aspects of child language are indeed adult-like from very early on, as Roger Brown recognized long ago (1973, p. 156). A highlight was the near-perfect correlation between the position and inflection of the verb in the main clause (Pierce 1992, etc.), which poses significant challenges for adult L2 learners (White 1990). At the same time, the problem of how children learn their specific grammars was left unaddressed.

On top of this, and perhaps because of it, there was a widely held belief that the commitment to Universal Grammar, shared broadly by both sides of the competence/performance divide, is inherently incompatible with input effects in language acquisition (Tomasello e.g., 2003, p. 97, Hoff 2014, p. 106). Distributional learning from data was viewed as evidence against Universal Grammar; see, for instance, "learning rediscovered" (Bates and Elman 1996) after the discovery of statistical learning for word segmentation (Saffran et al. 1996; see also Yang 2004). The same period also witnessed the so-called English past tense debate (Pinker and Ullman 2002, McClelland and Patterson 2002). Here the disagreement was on the treatment of the regular verbs — whether it was associatively based or whether it was handled by a rule that adds "-ed". For both sides, irregular past tense was formed by associative memory thus sensitive to frequency effects, apparently incompatible with the symbolic treatment of irregulars throughout the history of linguistics (Bloch 1947, Chomsky and Halle 1968).

2.2 The Variational Model

It was in this context that I proposed the variational learning model (Yang 2002). It was an acknowledgment that input effects matter for language acquisition but are completely consistent with the theory of Universal Grammar. It was also a return to the formalist tradition of language research. Rejecting the dominant view that the child language is characterized by a single grammar (e.g., an adult-like grammar, or a grammar in the space of possible grammars as in the Principles and Parameters framework and Optimality Theory), the variational model assumes that the grammars in the child's hypothesis space are associated with probabilities or weights. Learning takes place not by changing one grammar to another (e.g., Wexler and Culicover 1980, Berwick 1985, Gibson and Wexler 1994) but as changes in the probabilistic distribution of the grammars in response to input data. The simplest instantiation of the variational model is the Linear Reward Penalty scheme (Bush and Mosteller 1951), one of the oldest and best supported models from mathematical psychology.

For the purpose of illustration, consider a learner who has access to two grammars, the target A and a competitor B, which are currently associated with probabilities p and q. Upon encountering an input item s, the learner selects a grammar with its associated probability. Suppose A is chosen:

(1) a. If A can analyze s then $p' = p + \gamma q$ and $q' = (1-\gamma)q$
    b. If A cannot analyze s then $p' = (1-\gamma)p$ and $q' = q + \gamma p$

The chosen grammar has its probability increased if successful and decreased if not: in a zero-

guage variation (e.g., Gerken 1994, Demuth 1996), but these accounts also assert the correctness of child's competence grammar.

sum game, its competitor's probability adjustment is just the opposite. Several remarks about the variational model are in order.

First, the competition scheme in the variational model, inspired by evolutionary models of biological change (Yang 2006), implies some notion of fitness, and it is the fitness differential of the grammars that drives learning. In the simplest case, the target grammar A by definition always succeeds: p will rise whenever A is selected. The competitor B, by definition, must fail on a certain proportion of the input: when that happens, q will decrease and p will thus increase. But importantly, B needn't fail all the time: there may be input items that are ambiguous between the grammars. Thus, the trajectory of learning in the long run is determined by the statistical composition of the input data. A grammar whose competitor is penalized more often will be learned faster.

Second, the grammar-input compatibility, referred to as "analyze" in (1), can be flexibly defined as long as it is precise and independently motivated. The simplest case would be parsability (e.g., whether the grammar is compatible with an input string) but many other considerations are possible. For instance, if the child's parsing system has certain limitations (Trueswell et al. 1999), then even sentences compatible with the target grammar may fail to be analyzed. The fitness values may also socially conditioned: a stigmatized variant would put its competitors at an advantage.[2]

Third, the variational model does not require that the hypotheses in competition are innately available. In fact the formalism is applicable to any finite set of hypotheses, including hypotheses that the learner constructs on the basis of specific language input. For instance, the model has been applied to word learning to represent the probabilistic association between the phonological form of a word and its meaning (Stevens et al. 2016), both of which are clearly learned from the environment. However, the explanatory value of the variational model for specific cases of language acquisition lies in the specific hypotheses under competition, which may provide direct evidence for the nativist position as I discuss later in this section.

Finally, the variational model leaves space for individual variation. The statistical composition of the input may vary such that the target grammar may develop along different schedules for individual learners. It is also possible that some children are just slower at absorbing linguistic input than others; this is operationalized by the learning rate parameter γ in (1), which represents the magnitude of probability adjustment as the result of analysis, again a familiar notion from the mathematical psychology of learning. It has been suggested that individual variation in γ is a source for developmental language delay (Legate and Yang 2007).

The variational learning model was originally applied to the problem of parameter setting: for A and B in (1), think of the opposite values of a parameter. The model provably converges on the target grammar in the limit (Straus 2008). In a complex domain of thirteen word-order parameters (Sakas and Fodor 2012), the variational model has been shown to converge on the target consistently and efficiently (Sakas et al. 2017). More important, the variational model resolves several major challenges associated with traditional approaches to parameter setting. Its probabilistic nature means that the target grammar will only gradually rise to dominance under the cumulative effect of unambiguous data in its favor. Two empirical consequences follow.

[2] All the same, it is important to recognize that the fitness value, e.g., the probability with which a grammar fails to analyze the input data, is *not* something the learner needs to calculate — no more than the mouse needs to tabulate the probabilities of receiving food pellets in conditioning experiments (Bush and Mosteller 1951).

First, it is possible to establish the amount of unambiguous evidence for parameter values in child-directed input corpora to correlate with the developmental time course of the parameters. For instance, languages differ in the positioning of the main verb in the matrix clause: for languages like English, the verb follows adverbs (e.g., *John often drinks coffee*) whereas for languages like French, the verb precedes adverbs (e.g., *Jean boit souvent du café*). Only sentences that contain positional signposts such as the adverb *often/souvent* can unambiguously nudge the learner toward their language-specific option (White 1990); see Yang (2012) for a review of such input effects of parameters across languages. Recent work has devised intervention strategies (Hadley and Walsh 2014, Hadley et al. 2017) to boost English children's development of the morphosyntax of tense by amplifying the volume of informative data such as third-person singular present tense verbs in the caretaker input (Legate and Yang 2007, Yang et al. 2015).

Second, under the variational model, children's systematic deviation from the target grammar may be attributed to non-target hypotheses before their eventual demise; see Crain et al. (2016) for a recent review. Naturally, this perspective is only as good as what we take to be the space of linguistically possible hypotheses available to a child. While few would claim that human languages can vary arbitrarily, there is still considerable debate whether such constraints are specific to language or result from the constellation of other cognitive factors. At the same time, the theory of parameters is currently under review even within generative linguistics, especially in light of evolutionary considerations: that properties of human language previously thought to be domain specific may be ultimately grounded in other cognitive and perceptual systems (Hauser et al. 2002) and the principle of efficient computation (Chomsky 2005, Berwick and Chomsky 2016, Yang et al. 2017). In the concluding section of this paper, I will briefly discuss how the Tolerance Principle (Yang 2016), which is reviewed in Section 4, reduces the explanatory burden traditionally placed on innate linguistic parameters.

In my opinion, a theory of language acquisition need not be overly bound to the latest theory of linguistic structures. Child language can often be fruitfully studied as the level of empirical generalization which, if sufficiently robust, cannot only withstand the changing theoretical perspectives but also actively constrain them. Here I review one specific line of evidence uncovered through the variational model.

2.3 Null Subject: the Last Parameter?

Consider the classic problem of null arguments in child English (Bloom 1970). English-learning children frequently omit subjects — up to 30% of the time — and they occasionally omit objects as well, quite contrary to the input data they hear. (2) provides some naturally occurring examples from the CHILDES database.

(2) _ want cookies.
Where _ going?
How _ wash it?
Erica took _.
I put _ on.

These missing arguments generally do not impede language understanding. Nevertheless, children do not start using subjects and objects consistently at adult level around the third birthday.

Earlier attempts to equate the null argument stage to parameter missetting to the pro-drop or topic-drop option (Hyams 1986, 1991) were unsuccessful: during no stage of acquisition does the distribution of English-learning children's argument use resemble that of speakers or learners of pro-drop and topic-drop languages (Valian 1991, Wang et al. 1992), nor is there any evidence for sudden changes in the frequency of null subjects which would have supported the notion of parameter resetting (Bloom 1990, Legate and Yang 2007).

The variational model offers a new perspective on the null argument phenomenon. First, one needs to find distributional evidence in child language for the non-target grammars not yet unlearned. Second, one needs to quantify the disambiguating data in the input that eliminates these non-target grammars. More specifically, when English-learning child probabilistically accesses the target grammar, they will not generally omit the argument. But when the topic-drop grammar is accessed, argument omission would be possible when the discourse conditions are met. The most telling evidence can be found in a striking distributional property in child English. It is easy to find hundreds of child English examples in the CHILDES corpus of the following type:

(3) a. When _ bring the bag back? When _ rains?
b. Where _ get these? Where _ go? Why _ go slowly?
c. Why _ get scratched by the cat? Why _ laughing at me?
d. How _ fix my eye? How _ do open it?

These questions have target-like fronting of the wh-word but the subject is missing. Notice that these are all *adjunct* questions with *when*, *where*, *why*, and *how*. By contrast, omitted subjects in *argument* wh-questions are vanishingly rare:

(4) a. *Who$_t$ _ kissing t?
b. *Who$_t$ _ see t?
c. *What$_t$ _ want to hit t?

That is, when the object (*who* and *what*) is fronted in a wh-question, the subjects are almost never omitted. An exhaustive search of wh-questions produced by Adam, a prolific subject dropper, reveals a near categorical asymmetry (Yang 2002, p. 120):

(5) a. 95% (114/120) of the wh-questions with an omitted subject are adjunct questions.
b. 97.2% (209/215) of the wh-object questions contain subjects.

The null subject asymmetry in English-learning children's argument and adjunct questions is exactly mirrored in topic-drop languages. Consider the contrast between (6a) and (6b) in Mandarin Chinese. In both cases, suppose the existing discourse topic is the subject "John" but a new topic has been introduced via topicalization (in italic and marked with trace). The omission of the subject via discourse linking is only possible if the new topic is of a different type, namely an adjunct (6a) but not an argument (6b).

(6) a. *Mingtian$_t$*, [_ juede [t hui xiayu]]. (_ = John).
Tomorrow, [_ believe [t will rain]].
'It is tomorrow that John believes will rain.'

b. *$Bill_t$, [_ juede [t shi laoshi]]. (_ = John).
Bill, [_ believe [t is teacher]].
'It is Bill that John believes is the teacher.'

Even more direct parallelism comes from Brazilian Portuguese, a language that has overt movement in wh-questions (like English) but omits arguments in certain contexts (like Chinese).[3]

(7) a. Quando/Como/Onde$_t$ _ beijou t?
When/How/Where$_t$ _ kissed$_{2/3P}$ t?
'When/How/Where$_t$ did you/they kiss?'

b. *Quem$_t$ _ beijou t?
Whom$_t$ _ you/they kiss t?
'Whom did you/they kiss?'

The verbal morphology of Brazilian Portuguese has become too impoverished to support the agreement-based pro-drop option in its European cousin, as can be seen in the inflectional form of *beijou* in (7). Topic drop á la Chinese is the only option, and we see the exact asymmetry between adjunct and argument wh-questions — in child *English*.

Some may object to calling the topic-drop grammar a *parameter* but what's at stake is surely not terminological. The main generalization is that English-learning children spontaneously exercise a grammatical option never attested in their environment but used by speakers thousands of miles away. This option has to be suppressed by language-specific data: namely, the use of expletive subjects such as *There is a car coming* and *It seems that the kids are tired* because they are not thematic, thereby only serving a formal requirement of the English grammar. For instance, where the grammatical subject *it* must be present for the English expression *It is going to rain*, the position can be empty in Chinese ('_ yao xiayu' or "will rain"; Wang et al. 1992). Expletive subjects needn't be assumed to be a trigger innately associated with the English-type grammar (e.g., Hyams 1986). All that's required is for children to understand that the expletive subject, which does not receive a thematic role, must be a formal requirement on the grammar that the grammatical subject position needs to be overtly filled. Because expletive subject sentences are infrequent, making up about 1% of child-directed input, the rise of the obligatory subject grammar is gradual, according to the variational model. And the topic-drop grammar will be exercised during the process, resulting in null subjects in child English as well as occasionally null objects, when the object happens to be the discourse topic.

Naturally, the same model ought to account for the acquisition of topic- and pro-drop languages (Valian 1991, Wang et al. 1992, Kim 2000, Grinstead 2000) which, in contrast to the considerable delay in English, show very early adult-like command of subject use. I summarize these findings in Section 5.1, where the variational model is extended to L2 acquisition of the subject.

[3] I thank Pablo Faria and Guilherme Garcia for the data reported here.

3 Rules vs. Storage

At the 2005 LSA summer institute, I organized a workshop called "Nuts and Core",[4] borrowing Culicover's (1999) term for linguistic idiosyncracies that cannot all be plausibly attributed to an innate grammatical core (Chomsky 1981). The questions posed to the participants, all prominent scholars in the generative vs. constructivist debate, were as follows:

(8) a. If the core is dispensed with, how does the learner go from specific constructions to general regularities in syntax (Tomasello 2003)? What kind of constraints are needed for learning to be efficient and successful?

b. If the core is to be maintained, how might one construe a principled theory that keeps the core and the nuts separate (Fodor 2001)? How does the setting of a parameter value tolerate exceptions?

The workshop was lively but ended in a state of impasse. A main point of contention was whether child language is abstract and productive or item-based and lexically conservative. In many ways, the debate resembled the earlier competence-performance dispute: both sides offer useful but only partial explanations of child language. Later in this paper I will turn to my own proposal of how children discover productive rules — and potentially a reconciliation of the two approaches — but first, an assessment of the empirical and theoretical claims is in order.

3.1 Assessing Usage-based Learning

The first point to make is that an early stage of lexically specific language is neither a novel observation nor a feature unique to the usage/construction-based approach (despite being its "central tenet"; Diessel 2013). Again, let's turn to the study of English past tense. A well-known pattern is the U-shaped developmental curve. Children's verbal inflection is initially conservative: very few regular verbs are consistently marked in past tense, and the irregular verbs, when marked, are marked correctly. This stage is followed by the emergence of overregularization errors, which we can observe in longitudinal records. For instance, Adam's transcripts started at 2;3; all irregular verbs were marked correctly until 2;11, when he produced the utterance "What dat feeled like" (Marcus et al. 1992, Pinker 1995). Since *feeled* cannot be attributed to the input, the error marks the elevation of "-ed" to the status of a productive suffix. Thus English past tense is a classic case of initial conservatism followed by productive generalization. The phenomenon was central to the past tense debate and especially the dual-route model developed by Clahsen, Marcus, Pinker, and Prince: all avowed nativists.

The second, and more important, point is empirical: the evidence for an initial item-based stage of child language has been overstated. For example, high frequency combinations such as "give me" (sometimes "gimme") have been interpreted as "unanalyzed" collocations and presented as evidence for a lexically specific stage of language development (Lieven et al. 1992, Tomasello 1992, 2003). These expressions are indeed statistically dominant but to conclude that they are item based requires more work. At a minimum, one needs to show that their frequencies are conspicuously higher than expected had the verb and the pronoun been combined independently. In this light, consider the transcripts of the Harvard children Adam, Eve, and Sarah (Brown 1973),

[4] https://linguistlist.org/issues/16/16-2050.html.

datasets that have been in the public domain for decades. Searching for the strings *give me*, *give him* and *give her* in the three children's production data shows that they appear 95, 15, and 12 times, for the ratio of 7.75:1.23:1. It is thus quite likely that when working with a relatively small child corpus, *give* is only paired with *me*—perhaps the reason behind "give me" as a paradigm case of item-specific learning. But another search shows that the frequencies of *me*, *him*, and *her* in these children's production data are 2,949, 484, and 375, or 7.86:1.29:1. In other words, the frequency of "give me" actually suggest that the verb and the object combine independently and productively.

Quantitative research in the usage-based learning literature does not seem to have developed, and assessed, a coherently formulated null hypothesis: namely, that child language is productive, or that child language is *not* usage-based, however a usage-based account is formulated (and it is generally vague; see Tomasello 2000b for suggestions). Consider three key case studies in Tomasello's influential paper *Do young children have adult syntactic competence* (2000a) and other publications that are often cited as evidence for usage-based learning:

(9) a. The Verb Island Hypothesis (Tomasello 1992). Most of the verbs and predicates in early child language are used with one or very few possible frames.

b. Limited morphological inflection (Pizzuto and Caselli 1994). Almost half of the verbs in child Italian were used in one person-number agreement form (out of six possibilities), and only 13% of all verbs appeared in four or more forms.

c. Determiner imbalance. Pine and Lieven (1997) find that only 20-40% of the nouns that have been used the determiner *a* or *the* are used with both, despite the general interchangeability of the determiners (e.g., *a/the dog*, *a/the chair*, etc.).

So far as I can tell, these claims have been presented without evaluating an alternative hypothesis. At a minimum, it would have been worthwhile to subject *adult* language to item-based claims. With respect to determiner use (9c), quantitative analysis of English print materials such as the Brown Corpus (Kučera and Francis 1967) and child-directed speech reveals comparable, and comparably low, combinatorial diversity as children (Valian et al. 2009), yet adults' grammatical ability is not in question.[5] I now review the second equation, a statistically rigorous assessment of what constitutes evidence for a productive grammar (Yang 2013a). Usage-based claims can only be established if the alternative, grammar-based, hypothesis can be rejected.

3.2 A Statistical Benchmark for Grammar

To develop a principled quantitative interpretation of language, we must take Zipf's Law (1949) into account. For reasons no one quite understands (Miller 1957, Chomsky 1958, Mandelbrot 1953; see Yang 2013b for an exposition), word frequency is inversely proportionally related to rank. Specifically, let there be N unique words in a corpus. For the r-th ranked word, its frequency f is C/r where C is some constant. Thus, its probability of use p can be expressed as:

[5] Similar observations hold for verb islands (9a) and inflectional morphology (9b); see Kowalski and Yang (2012) and Yang (2016, chapter 2). Again, no alternative hypotheses were rigorously tested.

(10)

$$p = \frac{C/r}{C/1 + C/2 + \ldots + C/N}$$
$$= \frac{1}{rH_N} \text{ where } H_N = \sum_{i=1}^{N} \frac{1}{i} \text{ is the } N\text{th Harmonic number}$$

Particularly useful here is the approximation of the Harmonic number (H_N) as $\ln N$ which considerably simplifies the calculation of the statistical test (and the productivity model presented in Section 4).

The most obvious feature of Zipf's Law is the long tail: most linguistic units such as words, and by extension, combinations of words, are rarely used even in very large corpora (Jelinek 1998). This suggests that the sparsity of syntactic combinations in children's early language, or indeed any linguistic sample, is inherent: It does not automatically support lexically specific learning; as I discuss presently, it may even support a rule-based grammar.

The statistical benchmark for grammatical productivity (Yang 2013a) incorporates Zipf's Law to approximate word probabilities and their combinations. Let's consider its application to the sparsity of determiner-noun combinations in child and adult language. Suppose we have a corpus of N (singular) noun types that appear in S pairs of *a/the*-noun combinations. The expected probability for the r-th ranked noun having been paired with both *a* and *the* is given below:

(11)

$$E_r = 1 - (1-p)^s - \sum_{i=1}^{2} \left[(\mathbf{f_i p} + 1 - p)^S - (1-p)^S \right] \text{ where } p = \frac{1}{rH_N}$$

Here f_1 and f_2 are the probabilities of the two determiners. The reader is directed to Yang (2013a) for mathematical details but the most important point about the equation in (11) is highlighted in boldface, where we multiply the probability of the noun (p) with those of the determiners (f_1 and f_2). This assumes that their combinations are statistically independent, that is, *not* lexically specific. If a sample of determiner-noun combinations has been generated by an abstract and productive rule, then the average diversity value calculated from (11) should closely match the empirical value, namely the percentage of the N nouns used with both determiners. As shown in Table 1, although the combinatorial diversity is quite low across both child and adult languages, it is statistically indistinguishable, using tests such as the concordance correlation coefficient test (Lin 1989) from the expected diversity under a rule where the combinations are fully productive.[6]

The method developed in (11), which has been independently replicated by other groups (e.g., Silvey and Christodoulopoulos 2016), has broader applicability, benefiting from the accuracy of Zipf's Law (10), or $p = 1/rH_N$. We can calculate the expected combinatorial diversity based only on the sample size (S) and types (N) appearing in the sample, without even knowing the identities

[6] As Table 1 also makes clear, the actual value of diversity does not tell us anything about the underlying productivity of the grammar: unlike the claims in the usage-based literature, a higher diversity value (such as Nina) does not mean a "more" productive rule than a lower diversity value (such as the Brown corpus); see Pine et al. (2013) for a recent example of this fallacy. The formal analysis of the variation and how the empirical data are not predicted by usage-based learning models can be found in Yang (2013a).

Table 1: Empirical and expected combinatorial diversity in L1 English (adapted from Yang 2013a)

Subject	Sample size (S)	Types (N)	Empirical	Expected
Naomi (1;1-5;1)	884	349	19.8%	21.8%
Eve (1;6-2;3)	831	283	21.6%	25.4%
Sarah (2;3-5;1)	2453	640	29.2%	28.8%
Adam (2;3-4;10)	3729	780	32.3%	33.7%
Peter (1;4-2;10)	2873	480	40.4%	42.2%
Nina (1;11-3;11)	4542	660	46.7%	45.1%
Brown corpus	20650	4664	25.2%	26.5%

of the words. In recent work (Goldin-Meadow and Yang 2017), the method has been applied to home signs, the gestural systems created by deaf children with properties akin to grammatical categories, morphology, sentence structures, and semantic relations found in spoken and sign languages (Goldin-Meadow and Mylander 1998). Quantitative analysis of predicate-argument constructions suggests that, despite the absence of an input model, home signs show the expected degree of combinatorial productivity. By contrast, the test has also been used to provide rigorous supporting evidence that Nim Chimpsky, the chimpanzee raised in an American Sign Language environment, never mastered the productive combination of signs (Terrace et al. 1979, Terrace 1987): Nim's combinatorial diversity such as "'give Nim" and "give me" falls far below the level expected of a productive rule (Yang 2013a).

I must be clear about what the determiner productivity study does and does not show. It demonstrates that, at least for one aspect of child language, combinatorial productivity is on full display from the earliest testable stage of multi-word combinations. Thus, the usage-based claim for a lexically specific grammar is not supported. Furthermore, it provides a methodological example of how to develop statistically rigorous assessments of language data, a point to which I return in Section 5. But I do not suggest that all aspects of child language are productive from the get-go: see the remarks about English past tense earlier. Even for the determiner system, what we have established is a *formal* aspect of the grammar, that (at least) two syntactic categories combine freely and productively. Other properties of the determiner system — for instance, semantic properties such as count and mass, and pragmatic properties such as specificity — may take a good deal of fine tuning: non-target forms such as "a hair", "a blood", "a dirt", etc. are not uncommon in the speech of children whose determiner-noun combinatorial productivity is not in doubt; in fact, the successful acquisition of the formal system of determiner use appears to help establish the count/mass distinction in child language (e.g., Gordon 1985, 1988). In addition, the English determiner system contains some truly idiosyncratic elements: for example, while (American) English does not use determiners with place names ("*the Chicago", "*the Brooklyn"), there is "the Bronx", which can only be lexically memorized.

The very early syntactic productivity of the determiner system raises important questions: *How* do children learn this particular rule of the English grammar so quickly? Here Zipf's Law poses a significant challenge. The sparsity of language entails that the caretaker's speech can never be fully saturated with linguistic combinations. Recall that the combinatorial diversity in

caretaker speech is equally low as in child speech (Valian et al. 2009): that is, only a small fraction of nouns that can combine with both *a* and *the* will do so in the input. So how does the child generalize a property that holds for a small subset of words to all words — as they apparently do in Table 1? To answer this question, we need to confront the problem of learning by generalization: How do children acquire productive rules from lexical examples?

4 Productivity and Exceptions

As Sapir remarked "all grammars leak" (1928, p. 38-39): the balancing act between rules and exceptions is one of the oldest problems in linguistics. While linguists can distinguish rules from exceptions by carrying out grammaticality judgments and reaction time experiments (e.g., Clahsen 1999), children face a more formidable challenge. Since rules and exceptions are defined in opposition of each other, children seem to face a chicken-and-egg problem, and it is one that needs to be resolved in a few short years, without supervision or feedback, all the while under the sparsity of data befitting Zipf's Law.

4.1 Two Kinds of Exceptions

It is useful to distinguish two kinds of exceptions to rules. One kind can be called *positive exceptions*. The English past tense system is such an example: children receive overt evidence for the exceptions against the rule, by hearing irregular past tense forms that do not take "-ed". This is a familiar problem. A great many approaches, ranging from generative linguistics (e.g., Aronoff 1976, p. 36) to connectionist modeling (Marchman and Bates 1994) to hybrid models (Pinker 1999), share the same underlying intuition: a rule must "earn" its productivity, in the sense that it must somehow overcome the exceptions. It is frequently observed that a productive rule ought to be the one that covers the most diverse range of items: indeed, "statistical predominance" is traditionally the hallmark for linguistic productivity (e.g., Nida 1949, p. 14). However, (12) provides some illustrative problems from morphology and phonology, which suffice to show that the solution is not so simple:

(12)
a. English past tense: A default rule is learned abruptly and results in overregularization, after a protracted stage of rote memorization (Marcus et al. 1992, Yang 2002).
b. English stress: The grammar of English stress (Chomsky and Halle 1968, Hayes 1982, Halle and Vergnaud 1987) is not trochaic with a list of lexical exceptions despite a vast majority of English words bearing stress on the first syllable (Cutler and Carter 1987, Legate and Yang 2013).
c. German noun plurals: A suffix ("-s") can be the productive default despite coverage of fewer nouns than any of its four competitors (Clahsen et al. 1992, Wiese 1996).
d. Russian gaps: Morphological categories need not and sometimes do not have a default, as illustrated by the missing inflections of certain Russian verbs in the 1st person singular non-past (Halle 1973). Such cases are far from rare (Baerman et al. 2010): the absence of the past tense for *undergo* and past participle for *stride* are the more familiar examples from English speakers (Pullum and Wilson 1977, Pinker 1999). For

discussion in the context of L1 acquisition, see the curious case of Polish masculine genitives (Dąbrowska 2001, Yang 2016).

By contrast, *negative exceptions* in language learning seem more paradoxical. These are the cases where children must learn that a rule does *not* apply across to items that it could have (e.g., the nature of "minor rules" of Lakoff 1970, a long-standing problem). The well-researched English dative constructions illustrate the nature of the problem clearly:

(13) a. John gave the team a prize. John gave a prize to the team.
b. John assigned the students a textbook. John assigned a textbook to the students.
c. *John donated the museum the painting. John donated the painting to the museum.
d. John guaranteed the fans a victory. *John guaranteed a victory to the fans.

The verbs *give* and *promise* can freely alternate between the double object construction and the *to*-dative construction. However, semantically very similar verbs such as *donate* can only appear in the *to*-dative construction, and *guarantee* is exactly the opposite. Because children do not receive negative evidence, how do they learn what not to say in their language? Here a lexically conservative approach cannot work: the productivity of these constructions is evident in child language (Gropen et al. 1989, Conwell and Demuth 2007) and can also be observed when they are extended to novel verbs with appropriate semantic properties: when the verb *text* appeared, its double object form was instantly available as in *I texted them the score*.

Problems such as the acquisition of the dative constructions once dominated the learnability research in the generative tradition (Baker 1979, Berwick 1985, Fodor and Crain 1987, Pinker 1989). In recent years, they have become a major focus of usage-based theories under the tenet of entrenchment (Tomasello 2003, Bybee 2006, Ambridge et al. 2008) or preemption (Stefanowitsch 2008, Boyd and Goldberg 2011): both are a form of indirect negative evidence which takes the absence of evidence as evidence of absence (Pinker 1989). According to a recent formulation (Ibbotson and Tomasello 2016), "if children hear quite often *She donated some books to the library*, then this usage preempts the temptation to say *She donated the library some books*." But use of indirect negative evidence is problematic (Pinker 1989): the absence of evidence is *not* evidence of absence. As I have argued elsewhere Yang (2015b, 2017) using realistic child-directed data, it is generally impossible to distinguish ungrammatical sentences, which would never appear in the input, from grammatical ones which just happen not to be sampled — because the space of linguistic combinations is enormous, and their statistical distribution is inherently sparse as implied by Zipf's Law. More empirically, such approaches fail to account for some of the most robustly attested errors in child language such as "I said her no" (Bowerman 1982, Bowerman and Croft 2008). The communication verb *say*, of course, is always used in the *to*-dative construction, and is among the most frequently used verbs in English — yet this (deeply) entrenched form fails to preempt the double object construction.

4.2 The Tolerance Principle

The Tolerance Principle and its corollary the Sufficiency Principle (Yang 2016) provide a unified solution for the problem of rules and exceptions. I will not review the empirical motivation for their development but will simply state:

(14) a. *Tolerance Principle*
Suppose a rule R is applicable to N items in a learner's vocabulary, of which e items do not follow R and are thus exceptions. The necessary and sufficient condition for the productivity of R is:

$$e \leq \theta_N \text{ where } \theta_N := \frac{N}{\ln N}$$

b. *Sufficiency Principle*
Suppose a rule R is applicable to N items in a learner's vocabulary, of which M follow R and no information is available about the remaining $(N-M)$ items. The necessary and sufficient condition for the productivity of R is:

$$(N-M) \leq \theta_N \text{ where } \theta_N := \frac{N}{\ln N}$$

The unifying theme of the two principles lies in the quantity of positive evidence — $(N-\theta_N)$ in both cases — that is necessary to support the productivity of a rule. To understand the intuition behind the rationale, consider an analogous case in a non-linguistic domain. Suppose you have encountered 10 new species off a remote island, of which 8 share a certain property (e.g., phosphorescence). Even though you may not have any information about the other two, or maybe even if the other two are known *not* to have the property, it seems reasonable to form a generalization about the entire class and extend it to the 11th species. By contrast, if the property only holds for 2 of the 10 examples, it seems wise not to rush to any general conclusion. The Tolerance Principle provides a precise weight of evidence, in the form of $\theta_N = N/\ln N$, that warrants productive generalizations.

The mechanism of learning under the Tolerance Principle is schematically illustrated in Figure 1.

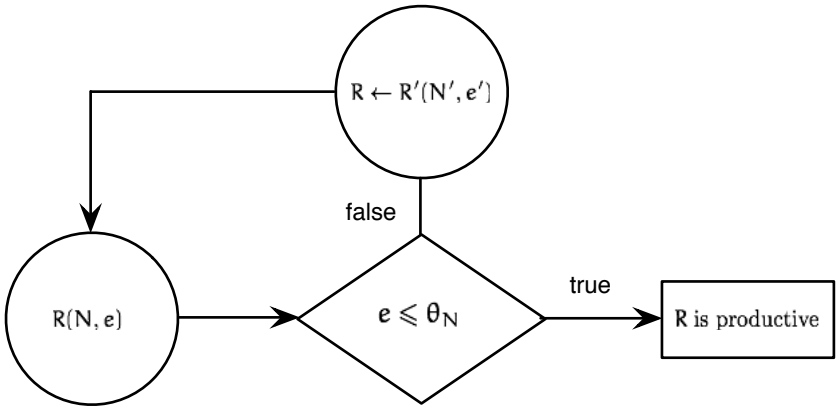

Figure 1: Tolerance Principle guides the search for productive rules in language learning.

Language learning is a search for productive generalizations that is best characterized as abductive learning (Chomsky 1968, p. 80). Children construct a rule R from the input data guided by linguistic and cognitive constraints and evaluate its productivity according to the associated numerical values (N and e). The rule is deemed productive if the positive evidence is sufficiently

high; otherwise learners formulate a revised rule (R') to obtain a new set of values (N' and e') and the Tolerance Principle is applied recursively. Thus, the quantitative accumulation of exceptions can lead to the qualitative change in the productivity of rules. An illustrative case study can be found in the analysis of English stress rules referred to in (12). When the child reaches a modest vocabulary (i.e., N is relatively large), the rule that stresses the first syllable fails to meet the productivity threshold despite covering the majority of words (Cutler and Carter 1987). The child subsequently divides the vocabulary into nouns and verbs, and productive rules within each subcategory can be established within (Legate and Yang 2013, Yang 2016); I will return to this theme in the context of multilingual acquisition in the final remarks of this paper. If no plausible rule can be found that meets the criterion for productivity, children will lexically memorize each instance that follows R and no productive generalization will be established.

Table 2 provides some sample values of N and the associate threshold values θ_N, the maximum number of exceptions that a productive rule can tolerate:

Table 2: The maximum number of exceptions for a productive rule over N items.

N	θ_N	%
10	4	40.0
20	7	35.0
50	13	26.0
100	22	22.0
200	38	19.0
500	80	16.0
1,000	145	14.5
5,000	587	11.7

These thresholds for productivity under the Tolerance Principle are significantly lower than a naïve "majority rule", which has many interesting implications for language acquisition. In particular, the Tolerance Principle asserts that a smaller vocabulary (i.e., smaller values of N) can tolerate a higher percentage of exceptions: all else being equal, productive rules are *easier* to detect for learners who have access to *less* input data. I return to this important theme in the conclusion.

Here I briefly summarize the application of the Tolerance and Sufficiency Principle to two of the most intensively studied problems in language acquisition: the past tense and the dative constructions in English.

In the case of English past tense, a distributional learning of induction must be used to identify the rules of verbal inflection. Many proposals of inductive learning, from many diverse fields (Chomsky 1955, Osherson and Smith 1981, Mitchell 1982, Cohen 1995, Yip and Sussman 1997, e.g.,), are applicable. These models typically operate by forming generalizations over exemplars in some suitable representation. For example, suppose two good baseball hitters can be described with feature bundles [+red cap, +black shirt, +long socks] and [+red cap, +black shirt, +short socks]. The rule "[+red cap, +black shirt] → good hitter" will follow, as the shared features (cap, shirt) are retained and the conflicting feature (sock) is neutralized. This method is thus very

capable of identifying the "-ed" suffix as one applicable without all phonological restrictions on the stem: the verbs that take "-ed" are phonologically very diverse, and no restrictions will be identified (Yip and Sussman 1997). Thus, the productivity of "-ed" will be determined by the total number of verbs (N) and the number of irregular exceptions (e) in the learner's vocabulary. The same consideration must also apply to the patterns that govern irregular verbs. For instance, the irregular verbs *bring*, *buy*, *catch*, *fight*, *seek*, *teach*, and *think* all undergo a stem change replacing the rime with [ɔt]. This rule also has no restriction on the stem, because the participating verbs are phonologically very diverse. But it is easy to see that the rule"rime →ɔt" will fare terribly: the seven positive members are easily swamped by hundreds of negative examples that do not change the rime to [ɔt], far exceeding the tolerance threshold. As a result, the rule is not productive and will be lexicalized — to these seven verbs. Other irregular patterns can be analyzed similarly: as shown elsewhere (Yang 2016, chap. 4), all rules except the regular "add *-d*" will be assessed as unproductive, accounting for the near-total absence of over-*ir*regularization errors in child English (Xu and Pinker 1995; see Lignos and Yang 2016 for a cross-linguistic review of similar findings.)

Following the same logic, we can see that the emergence of the "add *-d*" rule will require a long gestation period. Although children can quickly induce its structural description — perhaps using no more than a few dozen verbs (e.g. Yip and Sussman 1997) — irregulars are likely overrepresented in children's early vocabulary. For instance, in a 5-million-word corpus of child-directed English extracted from the CHILDES database (MacWhinney 2000), 76 of the 200 most frequent verbs in past tense are irregular. As θ_{200} is only 37, children with a small vocabulary are unlikely to establish the productivity of "-ed", despite the fact that it may be by far the statistically dominant rule. For very young children, then, verbs marked with "-ed" are in effect on par with irregulars: they are item-based and lexically memorized.

Telltale evidence for productivity comes from the first attested overregularization errors. When longitudinal records are available, the Tolerance Principle can account for the particular juncture at which productivity emerges. As noted earlier, Adam produced his first recorded overregularization error at 2;11 ("What dat feeled like?"). By then, Adam must have acquired a sufficiently large number of regular verbs to overwhelm the irregulars. In Adam's transcripts leading up to 2;11, he used $N = 300$ unique verbs in all, of which $e = 57$ are irregular. This is quite close to the predicted $\theta_{300} = 53$, and the discrepancy may be due to the underrepresentation of his regular verbs, which will be less frequent and thus more likely to be left out of a sample. Thus, Adam acquired a productive "-ed" only after he acquired a "super" majority of regular verbs, consistent with the Tolerance Principle.

4.3 Dative Generalizations and Retreats

The acquisition of the dative constructions is more complex as it involves children overgeneralizing before retreating: young children say "I said her no" but older children and adults do not. But the same procedure of constructing and evaluating generalizations applies here as well. Here I will briefly review the acquisition of the double object construction as the *to*-dative construction can be handled in a similar fashion (Yang 2016, chapter 6). In a five-million-word corpus of child-directed English data, roughly corresponding to one year worth of input, we can find a total of 42 verbs used in the double object construction. Of these, 38 have a very clear semantics of

"caused possession", which we assume is identifiable if the learner is equipped with a suitable set of conceptual and semantic primitives (e.g., Pinker 1989, Grimshaw 1990, Jackendoff 1990). The four exceptions, well below the threshold $\theta_{42} = 11$, do not convey caused possession: all are performative verbs (*call*, *consider*, *name*, and *pronounce*, e.g., *I called him a liar*). Thus, the semantic condition necessary for double object construction (Gropen et al. 1989, Pinker 1989, Levin 1993, Goldberg 1995, Pesetsky 1995, Krifka 1999) need not be stated as a UG primitive but can acquired from the language-specific data. The following hypothesis, then, can be formulated under the guidance of the Tolerance Principle:

(15) If a verb appears in the double object construction, then it will have the semantics of caused possession.

At this point, the child may consider the converse of (15), in trying to establish the validity of caused possession as a *sufficient* condition for the double object condition. This amounts to testing if the entire set of caused-possession verbs (N) in the child-directed corpus can be productively used in the double object construction. According to the Sufficiency Principle, this is warranted only if the $M = 38$ items, which are actually attested in the construction, constitute a sufficiently large subset of N.

In the present case, the input corpus contains an additional 11 verbs that belong to the semantic class in (15). But these did not appear in the double object construction:

(16) address, deliver, describe, explain, introduce, return, transport, ship, mention, report, say

(16) is an interesting list. For some of the items (e.g., *deliver* and *say*), the double object construction is ungrammatical: **John delivered the kids a pizza*, **John said Bill something mean.* But the verb *ship* does allow the double object construction — *John shipped Bill his purchase* — it just was not in the corpus because the caretakers opted for the *to*-dative form instead. Of course, the child does not know *why* the 11 verbs in (16) fail to show. The statistical distribution of language makes it difficult, if not impossible, to distinguish impossible forms from possible but unattested forms, which is the Achilles heel of indirect negative evidence and its entrechment or preemption variant (Yang 2015b, 2017). Nevertheless, $M = 38$ constitutes a sufficiently large subset of $N = 49$ since $49/\ln 49 = 12$, and thus the following generalization ensues:

(17) If a verb has the semantics of caused possession, then it can appear in the double object construction.

This immediately accounts for the overgeneralization errors such as *I said her no.* A search of child speech data in CHILDES also yields errors such as *I delivered you a lot of pizzas* (3;8), a verb not permissible in the construction in the adult language, thereby supporting the productivity of (17). This also accounts for the experimental evidence that children as young as 3;0 have productive usage of the dative constructions upon learning a novel verb with the appropriate semantic properties (e.g., Gropen et al. 1989, Conwell and Demuth 2007).

The retreat from overgeneralization straightforwardly follows the Tolerance/Sufficiency Principle but to do so requires the child to expand their vocabulary. Highly frequent verbs, which populate our child-directed corpus and are among those learned earlier by children, heavily favor the productive use of the rule in (17). I combined several corpora to approximate the vocabulary of ditransitive verbs likely known to most English speakers (Yang 2016, p. 208), and the results are

given in Table 3.

Table 3: Caused-possession verbs and their availability in the double object construction (adapted from Yang 2016).

top	yes	no	θ_N	productive?
10	9	1	4	Yes
20	17	3	7	Yes
30	26	4	9	Yes
40	30	10	11	Yes
50	34	16	13	No
60	39	21	15	No
70	43	27	16	No
80	46	24	18	No
92	50	42	20	No

Table 3 shows that a child with a very limited vocabulary is bound to conjecture (17) as a productive rule, namely, *all* caused-possession verbs may appear in the double object construction. But as they learn more verbs, the proportion of those used in the construction will continue to drop, eventually below the sufficiency threshold. The child can thus successfully retreat, without the problematic use of indirect negative evidence.

It is interesting to probe the properties of the verbs in Table 3 further, which provides a learning-theoretic account for many regularities in the double object construction and its acquisition (Mazurkewich and White 1984, Gropen et al. 1989, Bley-Vroman and Yoshinaga 1992, Inagaki 1997); see Yang and Montrul (2017) for a review. For example, 50 of the 92 verbs in Table 3 are monosyllabic, of which 42 allow double objects. By comparison, only 10 of the 42 polysyllabic verbs can participate in the construction. Thus, when given a novel verb that describes, say, the movement from an object initiated by one individual to another, both children and adults are more inclined to accept a short novel verb (e.g., *pell*) in the double object construction than a long one (e.g., *orgulate*). These tendencies are simply the consequences of distributional learning, rather than structural constraints as proposed in some theoretical literature (e.g., Harley and Miyagawa 2016). Similarly, while the entire class of caused-possession verbs cannot categorically participate in the double object construction, productivity can be found in semantic subclasses assuming that such classes can be constructed by language learners (Pinker 1989, Yang 2016): recall that productivity is more likely with smaller values of N.

In sum, the Tolerance Principle appears to embody what some usage-based researchers envision as the key solution to the problem of language learning: "a single mechanism responsible both for generalization, and for restricting these generalizations to items with particular semantic, pragmatic, phonological (and no doubt other) properties (Ambridge and Lieven 2011, p. 267)". Its simplicity yields sharp behavioral predictions, with interesting implications for the apparent differences between the outcome of L1 and L2 acquisition.

5 Formal Applications to L2 Acquisition

The mathematical models reviewed here address the type of questions that arise for language acquisition quite generally — by children and adults alike. Here I offer some comments and speculations on how these methods may apply to adult language acquisition.

5.1 UG Access and Input Frequency

Any learning model must consist of precise statements about the hypothesis space that the learner entertains – the initial state – and the mechanisms of learning from data (Chomsky 1965, Yang 2002). A successful model is the combination of the initial stage and the learning mechanisms that explains the specific patterns of language development. These questions are often debated in L1 acquisition but they arise for L2 as well, often in the form of the role concerning UG (e.g., Clahsen and Muysken 1986, Cook and Newson 2014, Epstein et al. 1996, Schwartz and Sprouse 1996, White 2003, Rothman and Slabakova 2017). Adult language learners also have an initial state, which may be partly dependent on their first language, and they also need to integrate data and experience into their linguistic knowledge. I would like to suggest that the variational learning model provides a useful perspective on these theoretical issues.

It is now evident that language, and language learning, obey certain structural constraints: not all formal language-like systems are natural or naturally learnable for by otherwise capable children and adults alike (Crain and Nakayama 1987, Smith and Tsimpli 1995, Newport and Aslin 2004, Tettamanti et al. 2004, Friederici 2017). What remains controversial is the nature of such constraints: what they are and whether they reflect domain specific restrictions on language or follow from more general cognitive principles. If it can be established that the variational model is employed by child and adult language learners alike, essentially by holding the learning mechanism component of the model constant, then the nature of the initial state may be fruitfully investigated.

The variational learning model is very likely implicated in language acquisition throughout the lifespan. In fact, it would be highly surprising if it were *not*, given its ubiquity in probabilistic learning and decision making across domains and species (Estes 1950, Bush and Mosteller 1951, Herrnstein and Loveland 1975), including language (Labov 1995, Roberts 1997, Hudson Kam and Newport 2005, Smith et al. 2009, Miller and Schmitt 2012). And there appear to be strong continuities between children and adults in at least certain domains of language. For word learning, the parallels between children and adults are very strong (Markson and Bloom 1997, Bloom 2000), and models of lexical acquisition are frequently tested on adult participants (Yu and Smith 2007, Stevens et al. 2016, e.g.). Furthermore, the model converges to a statistical combination of multiple hypotheses in linguistically heterogeneous environments as in the case of bilingualism and language change (Yang 2000), which further supports its broad applicability (see e.g., Slabakova 2008). Although there is no denying that adult, non-native grammatical acquisition is different from that of children in path and ultimate attainment, the differences do not have to relate to differences in underlying (cognitive/linguistic) mechanisms available to each. Let us assume, then, that variational learning is indeed used by language learners at all stages of development and explore how it helps determine the role of UG in adult language acquisition.

Consider, again, the obligatory use of grammatical subjects in languages such as English. As reviewed earlier, L1 acquisition can be modeled as a competition among the options delimited

by UG: the telltale evidence is frequent occurrence of null subjects in adjunct Wh questions (3) and near absence in argument Wh questions (4), both characteristic of a topic-drop grammar. How would an adult learner acquire a second language which uses the grammatical subject in a different way from their their first language?

In (18), I summarize the distributional evidence, extensively discussed elsewhere (Yang 2002), that can disambiguate the three broad classes of grammars from the perspective of the language learner. Their attested frequencies, as a percentage of the utterances in child-directed Chinese, Italian, and English are also reported.

(18) a. Chinese: Null objects (11.6%; Wang et al. 1992)
b. Italian: Null subjects in object wh-questions (10%; Yang 2002)
c. English: Non-referential expletive subjects (1.2%; Yang 2002)

The amount of unambiguous evidence for the Chinese and Italian type grammars is quite high. It is in fact, higher than the unambiguous evidence – about 7% (Yang 2002) – for the correct placement of the finite verb, which children acquire very early on and are essentially error-free (Pierce 1992). Thus, we predict very early acquisition of topic drop and pro drop. Indeed, studies of the acquisition of Italian (Valian 1991), Catalan/Spanish (Grinstead 2000), Chinese (Wang et al. 1992), Korean (Kim 2000), etc. have consistently found that children reach adult-level use of subjects around age 2, considerably earlier than the consistent use of grammatical subjects by English-learning children which typically takes place by age 3 or later.

Note that these findings about child language only follow if the initial stage consists of hypotheses, and their associated properties, laid out in (18). Because virtually all grammatical subjects are also thematic, the protracted null subject stage in child English must be linked to the low frequency of expletive subjects, which are the only type of input that distinguishes the competing options in UG. Suppose the initial state of the grammar is a probabilistic form of phrase structure rules such as $S \xrightarrow{p}$ NP VP, where p would be nearly 1.0 for English because the subject is almost always present and would be somewhere around 0.5 for Chinese (Wang et al. 1992), which allows topic drop. It is easy to see that if such rules were used, an English-learning children should quickly converge to the target grammar, contrary to the protracted subject drop stage in actual language acquisition.

With this background on L1 acquisition, we can turn to the question of adult second language acquisition. In fact, the published literature already points to strong parallels between children and adults: the acquisition of pro drop and topic drop appears "easier" than the acquisition of the obligatory subject use. For instance, Phinney's (1987) classic study finds that while advanced L1 Spanish learners of English do not consistently use expletive subjects, even beginner L1 English learners of Spanish show excellent command of pro drop; see also the later work of Pérez-Leroux and Glass (1999). The topic-drop option is likewise easily acquired. For example, Kanno (1997) finds that L1 English learners of Japanese have close-to-native command of null subjects across a number of syntactic and discourse contexts. By contrast, even near-native L2 learners of English fail to consistently use the expletive subject (Judy 2011), the true hallmark of the obligatory subject grammar. The variational learning model provides a straightforward account for these cross-linguistic findings. As shown in (18), the advantage of the topic/pro-drop grammars over the obligatory subject grammar is afforded by the more abundant disambiguating evidence in the input language, because variational learning is gradual, probabilistic, and quantity sensitive.

The variational approach to L2 acquisition can help make an even stronger claim about the role of UG. If adult language acquisition mirrors child language acquisition, then one must conclude that the parametric options of UG are available to children and adults alike, which amounts to a very strong form of UG access (White 1989, Schwartz and Sprouse 1994, 1996, Epstein et al. 1996, White 2003). To establish this claim requires the same kind of evidence from child language acquisition under the variational model: we need to find the footprints of non-target yet UG-consistent grammatical hypotheses. We thus turn again to the argument vs. adjunct asymmetry: null subjects are possible in adjunct wh-questions but not possible in argument wh-questions.

Such an asymmetry, if confirmed, would not be very surprising for an L2 learner whose L1 is a topic-drop grammar and transfers into L2, but it would provide strong evidence for the full accessibility of UG for learners whose L1 is a pro-drop language, which licenses null subjects by agreement and does not show this restriction. I am not aware of any study of adult acquisition of the English grammatical subject that specifically targets these distributional properties. In what follows, I can only offer a preliminary analysis based on a corpus provided by Klein and Perdue (Perdue 1993, available at talkbank.org).

There are four adult English learners whose L1 is Italian, which provide suitable opportunity to examine the distribution of null subjects in their wh-questions. I extracted all of their wh-fronted questions. There are 35 (object) argument questions, only one missing the subject ("what doing in here"). There are 72 adjunct questions, with 16 missing the subject; some examples are given below:

(19) where is?
why shouldn't?
how much cost?
why no have appuntamento (appointment) in the evening?
when come in the school.

The sample size is small but there is a statistically significant difference ($p = 0.01$) between the null subject rates for argument and adjunct wh-questions. The predictions here are straightforward and can be easily verified in future research with larger datasets. The findings are suggestive: adult learners have access to a UG option – topic drop á la Chinese – that is neither in their first (Italian) or second (English) language.

The continuity between child and adult language, if true, points to potential intervention strategies. Although second language learners of English are frequently reminded of the fact that English requires the subject, only expletive subjects truly serve the purpose of driving the learner toward the target form, much like the acquisition of English by young children. Because expletive subjects are relatively infrequent in language use, amplifying the amount of such input may result in accelerated acquisition of the subject, similar to the improvement in L2 speech perception and production under targeted input (e.g., Bradlow et al. 1999). In addition, different intervention strategies may be designed to probe the nature of adult learners' initial state. For example, if adult learners more rapidly acquire the obligatory use of English subjects on the basis of expletive subjects rather than merely lexical or pronominal subjects as in the rule $S \rightarrow$ NP VP, it would provide further evidence for the continuity between child and adult language acquisition and amplify the role of UG.

5.2 Rule, Productivity, and Vocabulary

The statistical test for assessing grammatical productivity can be ported straightforwardly to the study of second language: Do L2 speakers go through a stage where the grammar is lexically specific and lacks abstract generalization? Claims from the L1 study of usage/item-based learning Tomasello 2000a, 2003) appear to have been imported into second language research (Ellis and Larsen-Freeman 2009): low frequency and diversity of syntactic combinations are taken as evidence for lexically based prototypes and exemplars and the absence of a fully abstract system. As discussed in Section 3, these claims cannot be taken at face value unless they are evaluated against rigorously formulated statistical hypotheses, including the null hypothesis that language – of children and adults – is in fact fully productive.

In this section, I apply the methods developed by assessing child language to adult language. The inferential problem is the same: given a linguistic corpus, what is the nature of the underlying mechanism that generates the production? Is it a fully productive system, or is it lexically specific? The case study focuses on seven adult learners of English studied by Klein and Perdue (Perdue 1993, available at talkbank.org). The methods are very simple and can be automated with simple natural language processing tools (see Yang 2013a and Silvey and Christodoulopoulos 2016 for details).

Table 4: Empirical and expected combinatorial diversity in L2 English (data from talkbank)

Subject	L1	Sample size (S)	Types (N)	Empirical	Expected
Andrea	Italian	355	171	4.1%	9.8%
Lavinia	Italian	822	295	19.3%	22.6%
Santo	Italian	398	193	3.1%	9.6%
Vito	Italian	314	139	6.5%	11.0%
Ravinder	Punjabi	121	75	5.3%	8.6%
Jainail	Punjabi	283	148	8.1%	9.3%
Madan	Punjabi	237	102	3.9%	11.7%

The L2 learners' use of determiner-noun combinations is significantly below the diversity level expected under a fully productive grammatical rule ($p < 0.001$; paired one-tailed Mann-Whitney test). Note however the samples in Table 4 are considerably smaller than the datasets from L1 acquisition and the results must be taken with a grain of salt.[7] But Table 4 does appear to reveal a usage-based stage of L2 acquisition in which learners have not mastered a simple grammatical rule, which L1 learners command with ease at a very early age (Table 1). This conclusion is consistent with reports of protracted development of the determiner system in L2 acquisition (Liu and Gleason 2002, Ionin et al. 2008, Snape 2008).

To understand the discrepancies between L1 and L2 acquisition, we must first address how young children acquire the determiner-noun rule in the first place. Because the rule is language specific, it must be learned distributionally from the input data. An examination of the child-directed input data turns up an interesting puzzle — one which may shed light on why adult language learners struggle with grammatical rules.

[7] This is also a plea for wider dissemination of L2 data for public use.

Consider Adam, for the last time. He produced 3,729 determiner-noun combinations in his speech with 780 distinct nouns. Of these, only 32.2% appeared with both determiners, which is similar to the expected value of 33.7%; see Table 1. Adam's mother, whose speech was also transcribed in the same corpus, produced a diversity measure of 30.3% out of 914 nouns.[8] Even among the 469 nouns used at least twice, which provided opportunities to be used with both determiners, only slightly over half of them (260) did so. To appreciate the logic of learning, consider a baseball analogy: the interchangeablity of *a* and *the* for a noun can be viewed a batter's ability to switch hit (i.e., batting both left- and right-handed). Suppose a scout has been sent to evaluate a team of players, about a third of whom switch-hitted in the batting practice. It would seem crazy to conclude *all* players in the squad are switch-hitters, but that is apparently what Adam did: he generalized the interchangeability of *a* and *the* from a third of nouns to all nouns. Such an inductive leap seems absurd. It is certainly not sanctioned by the Tolerance/Sufficiency Principle, which asserts that a rule — the interchangeability of *a* and *the* — can be extended to a class of words only if the rule holds for an overwhelming majority of the words.

A promising, and perhaps the only, way out of this dilemma is to make use of a key property of the Tolerance Principle: rule learning is easier, and more tolerant of exceptions, when the learner has a smaller set of items in their vocabulary (Table 2). The developmental literature offers the idea of "less is more" (Newport 1990, Elman 1993, Kareev 1995, Cochran et al. 1999): the maturational constraints place a limit on the processing capacity of young children, which may turn out to be beneficial for language acquisition. If children's vocabulary is smaller, the odds of acquiring productive rules improve considerably.

Consider again the determiner-noun combinations produced by Adam's mother. Only 277 out of the 914 nouns are used with both *a* and *the*, which is nowhere near the requisite threshold for generalization ($\theta_{914} = 134$). But if Adam were only to learn from the 50 most frequent nouns, he would notice that almost all of them — 43 to be precise — are paired with both determiners. On this much smaller subset of data where $N = 50$, there is sufficient evidence for generalization: the 7 nouns that appear exclusively with only one determiner are below the tolerance threshold $\theta_{50} = 12$. For the top $N = 100$ nouns, 83 are paired with both determiners: the 17 loners are again below the tolerance threshold $\theta_{100} = 23$. At the time when children acquire the productive rule for determiners, their vocabulary size does not exceed a few hundred (Fenson et al. 1994, Hart and Risley 1995). It is thus highly likely that they have acquired the rule on a very small set of high frequency nouns, almost all of which will show interchangeability with both determiners; the rest is just noise.

This line of thinking naturally leads us to speculate why adults tend to be worse at language learning than children, when they are better at pretty much everything else. There are of course many differences between children and adults but I would put forward a simple but bold possibility. By the virtue of having greater cognitive capacities, or perhaps due to the influence of their L1 lexicon in developing an L2 lexicon (Jiang 2000, Dijkstra 2005, Van Assche et al. 2012, Tokowicz 2014), adult learners may have know too many words for their own good.

Table 5 gives the token/type ratio for the words used by the L1 and L2 learners, whose productivity measures have been given in Tables 1 and 4. The ratio provides a rough measure of the language user's vocabulary. A ratio of X means that, on average, the speaker produces a new word type every X words; thus, a small token/type ratio is an indication of a larger vocabulary, a

[8] Once again, this low diversity does *not* imply that Adam's mother does not use the determiner rule productively.

Table 5: Vocabulary size estimates and productivity in L1 and L2.

Subject	Token	Type	Token/Type
Adam	140,793	3,811	36.94
Eve	27,147	1,582	17.16
Naomi	35,459	2,274	15.59
Nina	87,933	2,553	34.44
Peter	62,006	1,936	32.03
Sarah	73,951	3,658	20.22
Andrea	8,097	1,013	7.99
Lavinia	16,941	1,673	10.13
Santo	10,705	1,284	8.34
Vito	9,344	1,296	7.21
Ravinder	9,980	954	10.46
Jainail	10,017	913	10.97
Madan	8,672	909	9.54

long-standing practice in language research (Miller 1981, Huttenlocher et al. 1991). It is evident that the adults have considerably larger vocabularies than the children.[9]

I suggest that the L2 learners's apparent inability to use a simple rule fully productively is because they know *too many* words. Young children have no choice but to learn from a small set of high frequency words for which the evidence for productive rules is sufficiently strong. Again, toddlers' vocabulary size has been estimated to be no more than just over a thousand (Huttenlocher et al. 1991, Hart and Risley 1995), yet children have perfect command of a productive system including the determiner rule reviewed earlier and many aspects of morphology and syntax (see Guasti 2004 for a review). Thus, a small vocabulary must be sufficient and, if I am correct, necessary, for the acquisition of the essential components of language. Adults, by contrast, are in fact handicapped by their considerably larger vocabulary size due to their mature cognitive capacities. A larger value of N has the inadvertent consequence of raising the threshold for productivity, thereby making rule learning much more difficult.

Finally, a word about the application of the Tolerance Principle in adult language acquisition. The Principle is a method by which the learner evaluates potentially productive hypotheses about language. That these hypotheses are abstractions from input data and are subsequently evaluated against (further) input data entails that a quantitative measure of the input data is absolutely crucial. At this point, it is worth emphasizing that the values of N and e pertain to the vocabulary composition of specific learners, which necessarily vary on an individual basis. Thus, some learners may discover productive rules before others, and there is also the possibility that the terminal state of individuals' grammars varies with respect to the productivity of certain rules; see Yang (2016, Ch. 4) for case studies. Obtaining precise quantitative measures of the input data is obviously much harder for adult language learners. But it may be easier to obtain more accurate offline vocabulary measurements for adults, who are likely to be more cooperative than

[9]The data is scanty but even at the early testing sessions shortly after their arrival (Perdue 1993), the adult learners' token/type ratios are still considerably lower than young children's.

toddlers. Ultimately it is the individual's *internalized* vocabulary that determiners the productivity of rules. Furthermore, it is possible to devise experiments where one can have precise control over the individual's vocabulary with the use of artificial language (Schuler et al. 2016, Schuler 2017).

5.3 Final Remarks

The mathematical models reviewed here, especially the variational learning model and the Tolerance Principle, are meant to complement each other. In this concluding section, I will first describe how these learning mechanisms interact in both L1 and L2 acquisition before making some general methodological remarks.

While making use of domain-general learning mechanisms, the variational model was developed in the "orthodox" framework of parameter setting, where the grammatical options are made available by an innate UG and subject to competition. Later developments in linguistic theories (Chomsky 2001, 2005, Berwick and Chomsky 2016) have aimed to reduce the principles specific to language, and the Tolerance Principle can be viewed as a move into that direction: what can be learned abductively from the linguistic data needn't be built into UG.

As discussed throughout the paper, I believe that at the present stage of understanding, certain grammatical options in language concerning the subject still appear unlearned. But as illustrated in the acquisition of the dative constructions (Section 4.3) and the determiner-noun rule in child English (Section 5.2), the Tolerance Principle offers an alternative account of word order phenomena that traditionally fall under the purvey of syntactic parameters such as head directionality. For example, that English prepositional phrases are head initial can be learned if the child keeps track of the positional relation between prepositions — a finite number of items — and their complements and forms a categorical generalization. Even a (suitably) low degree of exceptions can be tolerated. For instance, the vast majority of the verbs in English do not raise in question formation; those that do, namely a small number of auxiliary verbs, can easily be learned as an exception.

In some cases, the Tolerance Principle may not be able to identify a single productive rule for the totality of the input data and recursive applications will be required. The most radical case will be bilingual acquisition or transitional stages of language change that has been characterized as "grammar competition" (Kroch 1989). As suggested elsewhere (Yang 2016, p137), the problem of distinguishing that there are multiple languages in the input, a very first step in multilingual acquisition, is formally equivalent to the problem of distinguishing sub-regularities in a single language. A handful of strange words or visiting relatives with a silly accent will not disrupt the acquisition of a single linguistic system, just as a few irregular verbs do not undermine the regular "add -d" rule. But if the learner's environment consists of significant quantities of multilingual data, then no single language is likely to tolerate the others as exceptions. The learner will be compelled to partition the input into distinct subsystems and develop independent grammars for each, much like the acquisition of the English stress system, where the failure to establish a single productive rule for the entire vocabulary compels children to subdivide words into nouns and verbs and identify distinct stress rules within each class (Legate and Yang 2013, Yang 2016). Once multiple grammars are inductively established for a (sufficiently) heterogeneous body of language, the variational model can then apply to determine the course of their competition

(e.g., Yang 2000). This new way of probing the limit of experience and principles not specific to language — the second and third factor in the sense of (Chomsky 2005) — is a long term project and can be only be successful when we successfully account for the full range of linguistic facts, which traditionally fell under the first factor, Universal Grammar.

To conclude, it is useful to recall that the rise of modern linguistics and cognitive science was enabled by the formal methods introduced by generative grammar: abstraction and idealization over complex phenomena, followed by deductive analysis of nontrivial depth. Sixty years later, the worst one could wish for are proposals where all conceivable factors are thrown into a stew just so no correlation could ever be missed: that would be to return to the Dark Ages of standard social and behavioral sciences. These approaches do not enlighten but only obfuscate (Yang 2015a).

Much more research, and especially more data, will be needed to further test these mathematical models for L1 acquisition and to verify their applicability to L2 acquisition. All the same, I hope to have conveyed the importance of formal methods to the study of language acquisition. The equations may turn out to be wrong. But one of the most appealing aspects about language is that it is tractable, and even mechanical. We can make progress even by making mistakes so long as the mistakes are precisely formulated.

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