

(NON)-MAXIMALITY AND DISTRIBUTIVITY: A DECISION THEORY APPROACH

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1. Introduction

1.1 Vagueness in questions.

- Following Merin's (1999) approach to relevance, van Rooy (2003) - Decision Theory (DT) approach to questions.
 - questions are underspecified or vague (1):
 - variable Op (1b) depends on a decision problem facing the speaker.
 - In the context of the decision problem, vagueness is resolved (1d).

(1) a. Where do you live?

b. $\{\lambda v [g \in \text{Op}(P)(v)]: w \in W \& g \in \text{Op}(P)(w)\}$, with $P = \lambda w \lambda x \text{You live in } x \text{ in } w$

c. *Possible partitions:* i. {You live in USA, You live in France,...}
ii. {You live at 1 High St. Enid OK,...}

d. *Goal 1:* At LSA, questioner interested in background of hearer → partition i
Goal 2: At LSA, questioner has to mail hearer a package → partition ii

- The DT approach derives two main types of vagueness:
 - level of granularity as in (1),
 - and degree of exhaustivity (2), where the desired answer could be strongly exhaustive (mention-all) or mention-some.

(2) Where can I buy an Italian newspaper?

1.2 Vagueness in plurals.

- Parallels exist between the vagueness in questions and in definite plurals.
 - First, as repeatedly noted in the literature, definite plurals are vague w.r.t the level of granularity (distributivity) (3).

(3) a. The boys built a raft.

b. *Possible interpretations:*

i. $\text{Team}_1 = \{\text{Andy}, \text{Bill}\}$ built a raft and $\text{Team}_2 = \{\text{Chris}, \text{Dan}\}$ built a raft
ii. Andy built a raft & Bill built a raft & Chris built a raft & Dan built a raft

- Second, definites are vague w.r.t. exhaustivity:
(4) is compatible with the maximal (all the windows) or non-maximal construal of the definite, e.g. excepting a few closed windows if the hearer is wondering if his home is storm-proof (Krifka 1996).

(4) a. The windows are open.

b. *Possible interpretations:* i. All of the windows are open
ii. Some of the windows are open

- In resolving the vagueness, what factors affect the final interpretation for sentences with definite plurals?
 - First, the nature of the VP plays a role.
 - level of distributivity or collectivity in the subject (5a,b) (Dowty 1987),
 - preference for maximal (5c) or non-maximal (5d) interpretation (Yoon 1996).

- (5) a. The boys took a deep breath
 b. The boys surrounded the castle
 c. The children are healthy
 d. The children are sick

- VP doesn't fully determine the final interpretation – extra-linguistic factors play a role in resolving the vagueness in both distributivity and maximality.
 - Schwarzschild 1991 - influence of the situation on distributivity (6a, from Schwarzschild 1991);
 - Krifka (1996) - context over-riding VP influence (6b,c, from Krifka 1996).

- (6) a. Situation: Vegetables arrive at a grocery pre-packaged in baskets. The grocery has a big rough scale suitable for small truckloads, and a small scale for weighing only a few veggies or fruits at a time.

Utterance: The vegetables are too light for the big scale and too heavy for the small scale

- b. Situation: I am about to travel, and want to ensure my house is safe in my absence.

Utterance: I returned to the house because I thought I had left the windows/doors open

- c. Situation: The local bank has a safe that is accessible only through a hallway with three doors, all of which must be open to reach the safe.

Utterance: I could reach the safe because the doors were open.

- Note that every scenario in (6) describes
 - an actual salient arrangement of the entities within the plural denotation into 'packages' (veggies pre-packaged in baskets, windows on the outside of the house, doors in a sequence),
 - as well as the goals of conversational participants, as spelled out in (7).

- (7) a. Utterance: The veggies are too light for the big scale. (excludes collective)

Packaging: Vegetables arrive at a grocery pre-packaged in baskets.

Goal 1: To double-check the amount of veggies delivered → Utterance is False

Goal 2: To price veggies for sale in pre-packaged baskets → Utterance is True

- b. Utterance: The windows are open

Packaging: The house has a dozen windows in its outside.

Goal 1: To prepare the house for the arrival of window-frame painters → All windows

Goal 2: To ensure the house is rain-proof before a thunderstorm → Some windows

- The empirical data thus indicates the conditions for a theory of interpretation of definite plurals.
 - 1) A vague or underspecified semantics that allows for the full range of the collective/distributive & maximal/non-maximal variation.

- 2) A way to encode the effects of the VP, the packaging of entities within the NP denotation, and the speaker/hearer goals in computing the truth-conditions.

The idea that both packaging and speaker/hearer goals play a role in interpretation of definite plurals is fairly uncontroversial. Existing accounts, however, have either not spelled out the role of extra-linguistic factors, or focused on the role of packaging. In the next section, I will sketch three promising approaches to distributivity and maximality in definite plurals, and point to the need for an upgraded account.

2. Existing approaches.

2.1 Landman 1989.

- Building on the work of Link (1983), Landman (1989) offers a semantic theory that permits interpretations of varying distributivity and (non)-maximality.
 - Star (*) operator pluralises VPs, creating distributive interpretations,
 - Predicates distribute over sums ($a \sqcup b \sqcup c$), but not over groups ($\uparrow(a \sqcup b \sqcup c)$).
 - All non-maximal interpretations are instances of collective predication (where the plural is interpreted as a group).
- Thus, the interpretations for (5ab,6bc) arise from representations in (8), where Andy, Bill, Chris, and Dan are the boys, and d_1, \dots, d_5 are the doors (or windows) (after arrow is a simplified formula indicating a situation in which the interpretation is true).
 - Note that the only way to require a fully maximal interpretation is to make it fully distributive (8d).

- (8) a. $\llbracket 5a \rrbracket = \llbracket \text{The boys took a deep breath} \rrbracket =$
 $= \lambda w. \exists e \text{ *TOOK.A.DEEP.BREATH}(e, w) \ \& \ \text{*Ag}(e) = \sigma x \text{ *boy}(x, w) \rightarrow$
 $\rightarrow (a \sqcup b \sqcup c \sqcup d) \in \text{*TOOK.A.DEEP.BREATH}(w)$ **distributive**
- b. $\llbracket 5b \rrbracket = \llbracket \text{The boys surrounded the castle} \rrbracket =$
 $= \lambda w. \exists e \text{ *SURROUNDED.THE.CASTLE}(e, w) \ \& \ \text{*Ag}(e) = \uparrow \sigma x \text{ *boy}(x, w) \rightarrow$
 $\rightarrow \uparrow(a \sqcup b \sqcup c \sqcup d) \in \text{SURROUNDED.THE.CASTLE}(w)$ **collective**
- c. $\llbracket 6b \rrbracket = \llbracket \text{The doors are open} \rrbracket =$
 $= \lambda w. \exists e \text{ *OPEN}(e, w) \ \& \ \text{*Ag}(e) = \uparrow \sigma x \text{ *door}(x, w) \rightarrow$
 $\rightarrow \uparrow(d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5) \in \text{OPEN}(w)^1$ **collective**
- d. $\llbracket 6c \rrbracket = \llbracket \text{The doors are open} \rrbracket = \lambda w. \exists e \text{ OPEN}^*(e, w) \ \& \ \text{*Ag}(e) = \sigma x \text{ door}^*(x, w) \rightarrow$
 $\rightarrow (d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5) \in \text{OPEN}^*(w)$ **distributive**

- It is not clear whether Landman's 1989 theory allows for the intermediate-distributive interpretations like (6a), repeated below. His 1996 paper derives them via adaptation of Schwarzschild's 1991 proposal described in section 2.3

¹ I am supposing here that 'open', unlike 'take a breath,' does not have a built-in lexical requirement that it must distribute fully down to individuals. Then, Landman 1989 and 1996 can derive non-maximal interpretations for 'open' by giving the windows or doors a collective responsibility for the state of being open. Otherwise, we will need some pragmatic mechanism to derive non-maximality, like the one in Brisson 1998 or the one I propose below.

(8e, where artichoke, brussels sprout, cauliflower, daikon, and endive are the vegetables. The definition of cover-agent ${}^c\text{Ag}(e)$ is given in section 2.3)

(8) e. $\llbracket 6a \rrbracket = \llbracket \text{The vegetables are too light for the big scale} \rrbracket =$
 $= \lambda w. \exists e \text{ } {}^*\text{TOO-LIGHT}(e, w) \ \& \ \text{}^c\text{Ag}(e) = \uparrow \sigma x \text{ veg}^*(x, w) \rightarrow$
 $\rightarrow (\uparrow(a \sqcup b) \sqcup \uparrow(c \sqcup d \sqcup e)) \in \text{}^*\text{TOO-LIGHT}(w) \quad \textbf{intermediate-distributive}$

- Landman’s theory posits a pervasive ambiguity between singular and plural VPs, and does not distinguish between the maximal and non-maximal interpretations in the representation (cf. 8b and 8c)².
- Even more importantly for us, as can be seen from (8), the theory does not provide an explanation of how and when the various interpretations arise, since it does not include an account of pragmatic factors (whether the structure of a situation or the intent of a conversational agent).

2.2 Schwarzschild 1991 and Brisson 1998.

- Schwarzschild (1991) proposes an account of distributivity, focussing on spelling out the role of packaging.
 - free variable over covers (Cov_i) in VPs allows them to distribute up to sub-pluralities of the definite plural given by the cover.
 - A cover is defined in (9a); the specific cover in (9b) derives the intermediate-distributive interpretations (6c,9c) from the representation in (9d).

(9) a. Cover: a set of sets of entities, s.t. the union of the sets in the cover is the universe of discourse.
b. $\{\{\text{artich1}, \text{artich2}, \text{Bruss.sprout1}, \text{Bruss.sprout2}\}, \{\text{caulifl}, \text{daikon}, \text{endive}\}, \{\text{John...}\}\}$
c. The vegetables are too heavy for the big scale and too light for the small scale
d. $\forall x [x \in \llbracket \text{Cov}_i \rrbracket] \ \& \ x \subseteq \llbracket \text{the.veg}' \rrbracket \rightarrow x \subseteq \llbracket \text{too-heavy-for-sc1} \ \& \ \text{too-light-for-sc2}' \rrbracket$

- Brisson (1998) builds on Schwarzschild (1991) to permit exceptions
 - by allowing mismatch between the distribution of individuals into the cover-cells and the NP denotations (ill-fitting covers (defined in 10a)).
 - Unlike Landman (1989), she only derives cases of a few salient exceptional items (10c, from representation in 10d), rather than existential interpretations like (6b,7b-2).

(10) a. A cover is *ill-fitting w.r.t. NP denotation* if some members of NP denotation are in the same cover-cell as non-members, so no union of cells in the cover equals the NP denotation.
b. $\{\{\text{window1}\}, \{\text{window2}\}, \{\text{window3}\}, \{\text{window4}\}, \{\text{window5}, \text{Mary}, \text{door}\}\}$
c. The windows are open (but we didn’t get to the bathroom window yet)
d. $\forall x [x \in \llbracket \text{Cov}_i \rrbracket] \ \& \ x \subseteq \llbracket \text{the.windows}' \rrbracket \rightarrow x \subseteq \llbracket \text{open}' \rrbracket$

There are serious drawbacks in the cover-based account.

² This inability to distinguish between maximal and non-maximal interpretations in theory matches empirical data for collective predicates: since the plural agent bears collective responsibility for the action, we cannot distinguish whether all or some of the agent’s subparts actually participated.

- 1) Final interpretation depends on the distribution of individuals into cover-cells, making non-maximality essentially a matter of narrowing the domain to a relevant set. In this, the cover-based account of Brisson 1998 inherits the problem of any theory that derives non-maximality through domain narrowing.
 - In fact, as examples (6b,7b) repeated below demonstrate, non-maximality is not a matter of domain selection of any kind:
 - there is no “relevant set” of open windows, and no window is more relevant than any other – it is simply irrelevant which or how many windows are open, as long as some of them are.

(6b) I returned to the house because I thought I had left the windows open.

- 2) The distribution of individuals into cover-cells is denoted by a free, deictic variable, whose denotation is fixed when a particular cover is made salient in preceding discourse or extra-linguistic context.
 - This deictic nature of Cov_i requires at least the speaker to know the exact composition of cells in the cover and in the special cell containing exceptions, in the same way this is required for a pronoun.
 - This is strikingly contrary to fact: the speaker doesn’t need to know the identity of the vegetables for the intermediate distributivity interpretation to arise in (6c,9c), nor does he need to know the identity of the exceptional windows for the non-maximal interpretation to obtain in (4,6b,7b).
- One way to avoid making demands on speaker knowledge is to intensionalise the Cov_i variable, varying the distribution of individuals into cells with each possible world compatible with what the speaker knows.
 - As Kratzer (2003) notes, “amending Schwarzschild’s account of plural predication, we would want to say that plural predication depends on contextually provided cover functions, not just on contextually provided covers.”
- The cover account derives
 - (some of) the influence of the VP (since the Cov_i variable is part of the VP),
 - and of the “packaging” aspect of scenarios (inasmuch as the situation always provides a salient cover),
 - but does not have a mechanism for integrating the influence of speaker/hearer goals into the analysis.

2.3 Landman 1996 and his recasting of Schwarzschild 1991.

- Landman (1996) provides a recasting of Schwarzschild’s 1991 proposal in his own framework (what he dubs ‘Theory IV’ of plurality).
 - This is done by allowing *cover roles*, defined as in (11a), and illustrated in the proof (11d) for the denotation (11c) for the sentence (11b), in a situation where Andy, Bill, Chris, and Dan are the boys (compare with 6c,9c).
- (11) a. Let R be a thematic role. Then cR , the *cover role based on* R , is the partial function from De (domain of events) into Dd (domain of individuals) defined by
- $${}^cR(e) = \mathbf{a} \text{ iff } \mathbf{a} \in \text{ATOM} \ \& \ \sqcup (\{ \downarrow(d) \in \text{SUM} : d \in \text{AT}({}^cR(e)) \}) = \downarrow(\mathbf{a}) \quad , \text{ i.e.}$$

a is either a singular atom or a group, and the set underlying **a** is the union of the elements underlying atomic subparts of the plural role based on R.

b. The boys built a raft. (in a situation where there are two teams of boys)

c. $\lambda w. \exists e \text{ *BUILT-A-RAFT}(e,w) \ \& \ \text{cAg}(e) = \uparrow \sigma x \text{ *boy}(x,w) \ \rightarrow$

$\rightarrow \lambda w. \uparrow(a \sqcup b) \sqcup \uparrow(c \sqcup d) \in \text{*BUILT-A-RAFT}(w)$

d. $e = f \sqcup g$, $\text{Ag}(f) = \uparrow(a \sqcup b)$, $\text{Ag}(g) = \uparrow(c \sqcup d)$; then $\text{*Ag}(e) = \uparrow(a \sqcup b) \sqcup \uparrow(c \sqcup d)$

$\{\downarrow(d) : d \in \text{AT}(\text{*R}(e))\} = \{a \sqcup b, c \sqcup d\}$, and

$\sqcup(\{\downarrow(d) : d \in \text{AT}(\text{*R}(e))\}) = \sqcup(\{a \sqcup b, c \sqcup d\}) = a \sqcup b \sqcup c \sqcup d = \downarrow(\uparrow \sigma x \text{ *boy}(x,w)) \ \text{qed}$

o The cover roles derive the interpretations in (5ab, 6abc) in a similar way (12).

(12) a. $\llbracket 5a \rrbracket = \llbracket \text{The boys took a deep breath} \rrbracket =$

$= \lambda w. \exists e \text{ *TOOK.A.DEEP.BREATH}(e,w) \ \& \ \text{cAg}(e) = \sigma x \text{ *boy}(x,w)$

$e = f \sqcup g \sqcup h \sqcup i$, $\text{Ag}(f) = a$, $\text{Ag}(g) = b$, $\text{Ag}(h) = c$, $\text{Ag}(i) = d$, $\text{*Ag}(e) = a \sqcup b \sqcup c \sqcup d$, so

$\sqcup(\{\downarrow(d) : d \in \text{AT}(\text{*R}(e))\}) = \sqcup(\{a,b,c,d\}) = a \sqcup b \sqcup c \sqcup d = \downarrow(\uparrow \sigma x \text{ *boy}(x,w))$

b. $\llbracket 5b \rrbracket = \llbracket \text{The boys surrounded the castle} \rrbracket =$

$= \lambda w. \exists e \text{ *SURROUNDED.THE.CASTLE}(e,w) \ \& \ \text{*Ag}(e) = \uparrow \sigma x \text{ *boy}(x,w)$

e is atomic, $\text{Ag}(e) = \text{*Ag}(e) = \uparrow(a \sqcup b \sqcup c \sqcup d)$, so

$\sqcup(\{\downarrow(d) : d \in \text{AT}(\text{*R}(e))\}) = \sqcup(\{a \sqcup b \sqcup c \sqcup d\}) = a \sqcup b \sqcup c \sqcup d = \downarrow(\uparrow \sigma x \text{ *boy}(x,w))$

c. $\llbracket 6b \rrbracket = \llbracket \text{The doors/windows are open} \rrbracket =$

$= \lambda w. \exists e \text{ *OPEN}(e,w) \ \& \ \text{*Ag}(e) = \uparrow \sigma x \text{ *door}(x,w)$ (see footnote 1)

e is atomic, $\text{Ag}(e) = \text{*Ag}(e) = \uparrow(d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5)$, so

$\sqcup(\{\downarrow(d)\}) = \sqcup(\{d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5\}) = d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5 = \downarrow(\uparrow \sigma x \text{ *door}(x,w))$

d. $\llbracket 6c \rrbracket = \llbracket \text{The doors are open} \rrbracket = \lambda w. \exists e \text{ OPEN}^*(e,w) \ \& \ \text{*Ag}(e) = \sigma x \text{ door}^*(x,w)$

$e = f \sqcup g \sqcup h \sqcup i \sqcup j$, $\text{Ag}(f) = d_1, \dots, \text{Ag}(j) = d_5$, thus $\text{*Ag}(e) = (d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5)$, so

$\sqcup(\{\downarrow(d)\}) = \sqcup(\{(d_1, d_2, d_3, d_4, d_5)\}) = d_1 \sqcup d_2 \sqcup d_3 \sqcup d_4 \sqcup d_5 = \downarrow(\uparrow \sigma x \text{ *door}(x,w))$

• Landman's reinterpretation preserves the flexibility of Schwarzschild's theory, allowing for the intermediate-distributive interpretations like (6c, 9c, 11d).

o The speaker only has to know that **there is a cover** making the statement true (i.e., the speaker must only be sure of the existence of a proof like the one in (11b), where knowledge that the teams are boy-made will point to the existence of such a proof).

o The analysis allows the full range of empirically attested interpretations, and says nothing about the way these various interpretations arise.

o Sentences like (13) just mean that subparts of the given group of windows/boys, in some packaging, are agents for (potentially plural) states of being open / events of raft-building.

(13) a. The windows are open

b. The boys built a raft

This semantics is very weak.

- The sentence with cover-roles just says that there is **some** proof like those in (11,12) –
 - *Prediction*: in a situation (6c), repeated below as (14), the speaker can utter ‘*The doors are open*’ on the **collective** interpretation (12c) that allows some of the doors to be actually closed, since a proof exists as long as **some** of the doors are open.

- (14) *Scenario*: The hearer works in a bank and must get to a safe that can only be reached via three consecutive doors.
Utterance: The doors are open
Interpretation: All of the doors are open
 - This is much less information than would actually be transmitted by such an utterance in the situation (14).
 - *Prediction*: in a situation (6a) repeated below in (15), the speaker can utter ‘*The vegetables are not too light for the big scale*’ on the **collective** interpretation – a true statement in a situation where the individual baskets **are** too light. After all, the possibility of putting all the baskets at once on the scale points to the existence of **some** way to make the sentence true.

- (15) *Scenario*: The vegetables arrive to the grocery store pre-packaged in baskets, and need to be priced by weight. The store has a scale suitable for truckloads of stuff.
Utterance: The vegetables are not too light for this scale
Interpretation: The veggies, weighed by the basket, are not too light for this scale.
 - Thus, by getting rid of the deictic cover variable, we have lost crucial information in this case.

- Too weak to stand on its own, this semantics is the perfect starting point for an account drawing on pragmatic factors to derive a stronger interpretation –
 - after all, in a particular scenario (6,7), hearers are perfectly clear on whether all or some windows/doors are required to be open, and on which packaging is meant when baskets of vegetables need to be weighed.

Interim conclusions:

In a theory of definite plurals, we must take into account that

- In accounting for the influence of **packaging** on interpretation,
 - Deixis to packaging is necessary in some cases (e.g., 6a,15) – without it, the semantics is too weak (14, 15)
 - Deixis to packaging is wrong in some cases (e.g., 6b, repeated below as 16) – the interpretations derived using it are contrary to fact.

- (16) *Scenario*: The house has a dozen windows on the outside. The hearer and speaker are going on a trip and want to make sure the house is safe.
Utterance: The windows are open
Cover-derived interpretation: The relevant/salient set of windows is open
Actually: Some of the windows are open (doesn’t matter/don’t know which)

- We still need to incorporate speaker/hearer goals into the analysis

In the proposal I lay out in the next section, I will introduce a deictic variable that will always include reference to hearer's goals, and encode information about packaging only when such information is relevant to hearer's goals.

3 Decision Theory approach

3.1 The proposal

- I apply Merin's (1999) and van Rooy's (2003) Decision Theory (DT) based definition of relevance to definite plurals, replacing the cover analysis.
- I integrate Landman's (1996) recasting of Schwarzschild's proposal with a principled account of how and when the various interpretations arise.
 - The chief innovation is thus the unification of the vague/flexible semantics with a formal analysis of pragmatic factors influencing the truth-conditions of sentences with definite plurals.
- Agents in conversation are constantly modeling each other's goals.
 - As a part of cooperative communication, each speaker aims to change the hearers' states of knowledge so as to help them progress towards their goals. This is the heart of the Cooperative Principle (Grice 1975).
 - When interpreting a vague utterance (in this case, one containing a definite plural), hearers select propositions which can influence their actions in achieving the goal.
- Agents' goals can be represented as decision problems (DeP) they are solving.
 - A DeP is a triple $\langle P, U, A \rangle$, where the probability function P represents agent's beliefs, utility function U reflects the agent's preferences, and a set of (mutually exclusive) actions A the agent chooses from.
- A proposition q changes agent's beliefs (P), resolving the DeP if, after q is learned, a single action has, in each resulting world, the highest utility.
- In making an utterance, the speaker aims to resolve hearers' DeP³.
- A relevance ordering between propositions (17) yields the contextual criterion for licensing and choosing an interpretation for plural definites, just as it does for questions, with relevance defined as helpfulness in resolving the DeP.

(17) Proposition p is more relevant (better to learn) for resolving DeP than q ($p \succ_{\text{DeP}} q$) iff
 i. p eliminates more actions as non-optimal than q does or
 ii. p eliminates the same number of actions as q does, and q entails p (i.e. q is over-informative)

- This relevance ordering, built into the definition of the variable REL (for 'relevant,' instead of van Rooy's Op)(18), allows the hearer to choose an appropriate interpretation for the vague definite.
 - REL replaces Schwarzschild's (1991) Cov_i in encoding the vagueness.⁴

³ Or, rather, the speaker aims to resolve his/her estimate or hearer's DeP. While practically all linguists agree that conversational participants represent each other's goals and knowledge during a conversation, the processes and representations utilized during such modeling are subject of much ongoing research. For the purposes of this paper and of (17), it is enough if speakers have an idea about the actions their hearers are choosing from, and can figure out the effect of learning various propositions on that action set.

(18) REL(DeP)(VP)(NP)(w)=

$$\{g: \boxed{\downarrow\{d: d \in AT(\downarrow g)\} \subseteq \downarrow NP(w)}\} \& \neg \exists h[\downarrow\{d: d \in AT(\downarrow h)\} \subseteq \downarrow NP(w) \& [\lambda w.VP(e,w) \& *Ag(e)=h \text{ } >_{DeP} \lambda w.VP(e,w) \& {}^c Ag(e)=g]]\}$$

- o When you boil **g** down to individuals, all of those individuals are atoms in the NP (e.g., “the boys”)
- o No other such NP-made thing h is such that it’s better to learn that “h VPs” than that “g VPs” – no h is more relevant than g

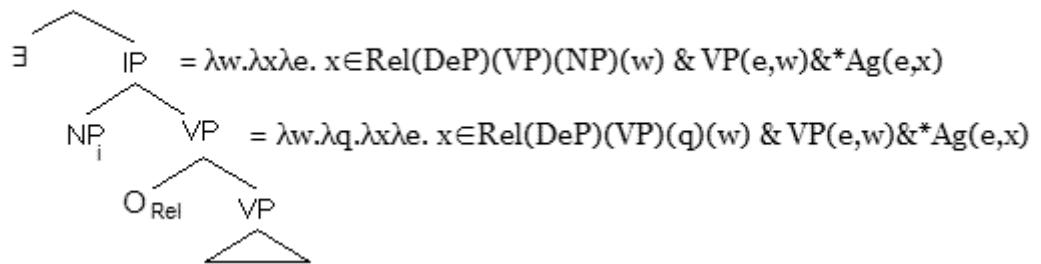
- REL operator comes as part of the VP, following syntactic assumptions in Schwarzschild 1991 and preceding literature on distributivity (19c).
- The operator containing REL introduces a set of alternatives – O_{Rel} is a function from VP and NP denotations to the set of optimally relevant propositions (19a).
- The sentential existential quantifier converts the set into a single proposition at the top (Hamblin semantics from Kratzer and Shimoyama 2002, see also earlier SALT 16 talk by Ezra Keshet).
 - o The schema for the entire sentence given in (19b).

(19)a. $O_{Rel} = \lambda w.\lambda P.\lambda q.\lambda x\lambda e. x \in Rel(DeP)(P)(q)(w) \& P(e,w) \& *Ag(e,x)$

b. $\lambda w.\exists p[p \in \{\lambda v.[g \in Rel(DeP)(VP)(NP)(v) \& VP(e,v) \& *Ag(e)=g]: g \in Rel(DeP)(VP)(NP)(w)\} \& p(w)=1]$

‘There is a true proposition, taken from the set of optimally-relevant propositions saying that some subpart(s) of the NP do the VPing.’

c. $\lambda w.\exists p[p \in \{\lambda v.[g \in Rel(DeP)(VP)(NP)(v) \& VP(e,v) \& *Ag(e)=g]: g \in Rel(DeP)(VP)(NP)(w)\} \& p(w)=1]$



3.2 Deriving (non)-maximality

Several worked-out examples illustrate the schema in (19).

(20) a. *Decision Problem*: Before a thunderstorm, Hearer has to decide whether to go on with daily business (*Action1* if all windows are closed) or return home (*Action2*) if some windows are open.

b. *Utterance*: The windows are closed=

$$= \lambda w.\exists p [p \in \{g(w) \in REL(DeP)(win)(clos)(w) \& g(w) \text{ is closed}\} \& p=1]$$

c. *Pool of propositions REL chooses from*:⁵

⁴ If x is an atom, $\downarrow x=x$ (Landman 1989). If x a sum, rather than an atom, I assume $\downarrow x=x$.

⁵ For clarity I simplify (c,d) keeping them extensional; VPs are (maybe vacuously) plural: *is closed* = *closed

$w1 \sqcup w2 \sqcup w3$ is closed	$\uparrow(w1 \sqcup w2 \sqcup w3)$ is closed
$w1 \sqcup w2$ closed $w2 \sqcup w3$ closed $w1 \sqcup w3$ is closed	$\uparrow(w1 \sqcup w2)$ closed $\uparrow(w2 \sqcup w3)$ closed $\uparrow(w1 \sqcup w3)$ closed
$w1$ is closed $w2$ is closed $w3$ is closed	$\uparrow(w1 \sqcup w2) \sqcup w3$ is closed ... $\uparrow(w1 \sqcup w2) \sqcup \uparrow(w1 \sqcup w3)$ is closed...

d. *Some possible interpretations:*⁶

- ⊙ i. $\exists p [p \in \{\exists e.*\text{clos}(e) \& w1+w2+w3+w4+w5=*Ag(e)\} \& p=1]$ **(all the windows...)**
- ii. $\exists p [p \in \{\exists e.*\text{clos}(e) \& w1=*Ag(e), \exists e.*\text{clos}(e) \& w2=*Ag(e), \dots\} \& p=1]$ **(some windows...)**

Consider an agent facing the problem in (20a).

- If he hears (20b), he has to select one of the interpretations [i] or [ii] as the true import of the vague literal statement.
 - While [i] resolves the decision problem (by pointing to *Action1*), [ii] fails to do so, since it is compatible both with a scenario where all windows are closed, and one in which some are open and some closed.
 - So, [i] is the only relevant interpretation.

(21)a. *Decision Problem:* Hearer is preparing his house for arrival of painters, who will paint all the window-frames. He has to decide if he can relax till they arrive (*Action 1*), if all the windows are open, or if he still needs to do something to prepare for the painters (*Action 2*), if some of the windows are closed.

b. *Utterance - same as in (20b):* The windows are closed=

$$= \lambda w. \exists p [p \in \{g(w) \in \text{REL}(\text{DeP})(\text{win})(\text{clos})(w) \& g(w) \text{ is closed}\} \& p=1]$$

c. *Pool of propositions REL chooses from* (see footnotes 5,6)

$w1 \sqcup w2 \sqcup w3$ is closed	$\uparrow(w1 \sqcup w2 \sqcup w3)$ is closed
$w1 \sqcup w2$ closed $w2 \sqcup w3$ closed $w1 \sqcup w3$ closed	$\uparrow(w1 \sqcup w2)$ closed $\uparrow(w2 \sqcup w3)$ closed $\uparrow(w1 \sqcup w3)$ closed
$w1$ is closed $w2$ is closed $w3$ is closed	$\uparrow(w1 \sqcup w2) \sqcup w3$ is closed ... $(w1 \sqcup w2) \sqcup \uparrow(w1 \sqcup w3)$ is closed...

d. *Some possible interpretations:*

- i. $\exists p [p \in \{\exists e.*\text{clos}(e) \& w1+w2+w3+w4+w5=*Ag(e)\} \& p=1]$ **(all the windows...)**
- ⊙ ii. $\exists p [p \in \{\exists e.*\text{clos}(e) \& w1=*Ag(e), \exists e.*\text{clos}(e) \& w2=*Ag(e), \dots\} \& p=1]$ **(some windows...)**

Now, consider an agent facing a different problem (21a).

- If he hears the first utterance (20b, 21b),
 - both [i] and [ii] resolve the DeP (by pointing to *Action2*) and thus are both relevant.
 - But [i] entails [ii] and thus is over-informative (hence less relevant according to (17)). This points to [ii] as the intended message.

3.3 Deriving distributivity

The worked example below illustrates that the hearer chooses the level of distributivity based on (17) in a (by now) familiar fashion.

⁶ This assumes that *closed/open* are not lexically distributive, i.e., they admit impure atoms like $\uparrow(w1 \sqcup w2)$. Otherwise, my account does semantically better than Landman 1989 (which loses the ability to derive non-maximal interpretations for sentences with ‘open’ and ‘closed’), REL has fewer propositions to choose from.

(22)a. *Decision Problem*: Hearer is deciding whether to pass the various boys' teams in a raft-building competition.

Action1: pass Team1, fail Team2. *Action2*: pass Team2, fail Team1. *Action3*: pass both, *Action4*: fail both.

b. The boys built a raft=

$$= \lambda w. \exists p [p \in \{g(w) \in \text{REL}(\text{DeP})(\text{boys})(\text{built..raft})(w) \ \& \ g(w) \text{ built a raft}\} \ \& \ p=1]$$

c. *Pool of propositions REL chooses from*:

$a \sqcup b \sqcup c \sqcup d \text{ built.raft}$	$\uparrow(a \sqcup b \sqcup c \sqcup d) \text{ built.raft} \approx$ $\text{Team1}=\uparrow(a \sqcup b \sqcup c \sqcup d) \sqcup \text{Team2}=\uparrow(a \sqcup b \sqcup c \sqcup d) \text{ built.raft}$
$a \sqcup b \sqcup c \text{ built.raft} \quad a \sqcup b \sqcup d \text{ built.raft} \quad \dots$	$\uparrow(a \sqcup b \sqcup c) \text{ built.raft} \approx$ $\text{Team1}=\uparrow(a \sqcup b \sqcup c) \sqcup \text{Team2}=\uparrow(a \sqcup b \sqcup c) \text{ built.raft} \dots$ $\uparrow(a \sqcup b) \text{ built.raft} \approx$ $\approx \text{Team1}=\uparrow(a \sqcup b) \sqcup \text{Team2}=\uparrow(a \sqcup b) \text{ built.raft} \dots$
$a \sqcup b \text{ built.raft} \quad b \sqcup d \text{ built.raft} \quad a \sqcup c \text{ built.raft} \dots$	$a \sqcup b \sqcup \uparrow(c \sqcup d) \text{ built.raft} \dots \quad \uparrow(a \sqcup b) \sqcup c \sqcup a \text{ built.raft} \dots$
$a \text{ built.raft} \approx \text{Team1}=a \sqcup \text{Team2}=a \text{ built.raft}$ $b \text{ built.raft} \approx \text{Team1}=b \sqcup \text{Team2}=b \text{ built.raft}$...	$\uparrow(a \sqcup b) \sqcup \uparrow(c \sqcup d) \text{ built.raft} \quad \uparrow(a \sqcup c) \sqcup \uparrow(b \sqcup d) \text{ built.raft}$ $\uparrow(a \sqcup b \sqcup c) \sqcup d \text{ built.raft}$ $\uparrow(a \sqcup b \sqcup c) \sqcup \uparrow(c \sqcup d) \text{ built.raft} \dots$

d. *Some possible interpretations*:

i. Andy built a raft and Bill built a raft, and Chris and Dan (as a group) collectively built a raft.

☺ ii. Team 1 built a raft & team 2 built a raft.

iii. Andy built a raft & Bill built a raft & Chris built a raft & Dan built a raft.

- Suppose that exact composition of the teams is not known
 - Interpretation [i] fails to resolve the question of the teams' performance.
 - Both the interpretations [ii] and [iii] resolve the problem, and so are relevant.
 - But [iii] is over-informative, and so [ii] is the interpretation of choice.
- The remaining propositions in the set represent the set of speaker's epistemic possibilities, i.e., the speaker's uncertainty about the composition of the teams.
- In an intensional framework, REL outputs functions $g(w)$ that, for each world, give the two boys' teams in that world.
 - The resulting set of propositions that remain after REL applies is then a singleton: {teams(w) built.raft}

4 Conclusions

4.1 Back to questions

- The DT approach takes as its base a weak semantics for definite plurals, and builds in pragmatic factors to derive stronger truth-conditions for sentences with definites.
 - Speakers' estimates for each other's beliefs, goals, and available actions are incorporated into selection of maximally relevant propositions with subparts of the plural definite (6,7).
 - The framework derives the (non)-maximality and distributivity patterns while making correct predictions regarding speaker/hearer knowledge.

- This framework makes interpretation of definite plurals parallel to that of questions (van Rooy 2003), where speakers' estimates for each other's Decision Problems are incorporated into selection of the set of maximally relevant answers.

(24) a. Where do you live?

b. $\{\lambda v[g \in \text{REL}(\text{place})(P)(v)]: w \in W \& g \in \text{REL}(\text{place})(P)(w)\}$, with $P = \lambda w \lambda x$ You live in x in w

c. Possible partitions: i. {You live in USA, You live in France,...}
ii. {You live at 1 High St. Enid OK,...}

d. *Decision Problem1*: At LSA, questioner interested in background of hearer to select a conversation topic: US sports (Action1), French cuisine (Action2), ...
Decision Problem2: At LSA, questioner has to mail hearer a package to (actions corresponding to different addresses)

(25) a. Where can I buy an Italian newspaper?

b. $\{\lambda v[g \in \text{REL}(\text{place})(P)(v)]: w \in W \& g \in \text{REL}(\text{place})(P)(w)\}$,
with $P = \lambda w. \lambda x$. Italian newspaper sold at x in w

c. Possible sets: i. $\{\{u = \text{It. news sold at the palace}\}, \{v = \text{It. news sold at the station}\},$
 $\{w = \text{It. news sold both at the palace and at the station}\}$
ii. $\{\{u, w\}, \{v, w\}\}$

Decision Problem1: In Amsterdam, questioner wants to read news in Italian by buying the newspaper at the station (Action1), or at the palace (Action2)

Decision Problem2: In Amsterdam, questioner is surveying availability of Italian press for a travel website (actions corresponding to writing down different sets of places where Italian newspapers are sold)

4.2 Overt distributivity and maximality operators

- Not all sentences with definite plurals are vague. Operators like *all* or *each* force maximal (and, in the case of *each*, fully distributive) interpretations in sentences with definite plurals (26), compare with (27).

(26) a. All the boys surrounded the castle.

b. The boys built a raft each.

(27) a. The boys surrounded the castle.

b. The boys built a raft.

- How do these lexical items achieve this effect? Two mechanisms are possible:
 - 1) The lexical operator (*all* or *each*) work in the semantics to limit or change the available interpretations.
 - 2) These operators contribute information about context (affecting the Decision Problem), so that REL, as a result, will give the desired interpretations.
- Winter (1998, 2002) adopts the former approach to these operators, while Brisson (1998) pursues the latter .
- Empirical data suggests that no extra-linguistic factors can force a maximal interpretation in collective sentences like (27a). Thus, it cannot be that *all* affects the context to force a maximal interpretation in (26a).
- This is matched by inability of REL, on our account, to distinguish between maximal and non-maximal interpretations for collective predication. Thus, I will adopt Winter's approach, in which *all* (and *each*) work in the semantics to introduce universal quantification (and full distributivity) over atoms in the NP.

- In Winter's (2002) framework NP and VP denotations can denote sets of atomic entities, or sets of sets of entities.
- He defines *all* as a composite determiner $dfit(every)$, where *every* has the usual denotation (a relation between sets of atomic entities), while $dfit$ is defined as in (28).

(28) Let D be a relation between sets of elements in a domain E of *atoms*. The operator $dfit$ (determiner fitting) maps D to a relation $dfit(D)$ between sets of sets of atoms in $p(E)$, which is defined as follows: for any two sets $A, B \in p(E)$, the relation $(dfit(D))(A, B)$ holds iff the relation $D(\cup A, \cup(A \cap B))$ holds.

- Translating this to the notation used here (with groups and sums), specifically for *all*, we get (29a). Without REL, this produces the denotation in (29b) for (26a)

(29) a. $all = dfit(every) = \lambda P. \lambda Q. \text{Every} (\sqcup \downarrow P, \sqcup (\downarrow P \sqcap \downarrow Q))$
 b. *all the boys surrounded the castle*

= $\text{Every} (\sigma x. *boy(x), \sqcup (\sigma x. *boy(x) \sqcap \sigma x. surrounded.castle(x)))$
 = Every boy participated in a set of boys that surrounded the castle

- As in the case without *all*, REL will produce a set of propositions in the end (because of the type mismatch, combining via function combination, as before). This will be a singleton set.
- Translating Winter's framework to the notation used in this paper, with some adjustments we will get (30)

(30) $\lambda Q. \text{Every} (\sqcup \downarrow NP, \sqcup (\downarrow NP \sqcap \downarrow Q)) (\lambda x. x \in \text{Rel}(\text{DeP})(\text{VP})(\text{NP}) \& \text{VP}(x)) =$
 = $\text{Every} (\sqcup \downarrow NP, \sqcup (\downarrow NP \sqcap \{\downarrow x: x \in \text{Rel}(\text{DeP})(\text{VP})(\text{NP}) \& \text{VP}(x)\}))$

This will result in the denotation for (26a) given in (31a) and paraphrased in (31b):

(31) a. $\text{Every} (\sigma x. *boy(x), \sqcup (\sigma x. *boy(x) \sqcap \sqcup \{\downarrow d: d \in \text{AT}(\downarrow x): x \in \text{Rel}(\text{DeP})(\text{sr.csl})(\sigma x. *boy(x) \& \text{sr.csl}(x))\}))$
 b. Every boy participated in a set of relevant boys that surrounded the castle.

Notice that (31b) is equivalent to (29b), except that (31b) can be taken to contribute an additional statement about the Decision Problem (namely, that the proposition with "all the boys" is optimally relevant).

4.3 Some further issues

- The Decision Theory approach, unifying the interpretation of interrogatives and definite plurals, lays the groundwork for explaining the parallel behaviour of questions and plurals with respect to QVE (Lahiri 1991, 1998, Williams 2000, Beck and Sharvit 2002) (32).

(32) a. For the most part, Al knows about which kids are drunk
 b. For the most part, Al hates the kids on his block
 c. For the most part, Al knows where the kids are hiding

- Quantification over parts can apply to wh-phrase denotations (32a) as well as to definite plurals (32b).
- Sometimes, it can apply to either, depending on the context (Williams 2000) (32c).

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