Potts (2005): Some More Math

LING 590, supplementary handout
March 2, 2009

1. VARYING N IN THE QUALITY FORMULA

1.1. Two countries, two cities per country, two streets per city

Last time, for the situation in which Q is neutral about where Barbara lives, and A knows that Barbara lives in Russia but doesn't know in which city, we had the chart:

A's belief	**Q's belief**	**Proposition: Barbara lives...**	**Quality-rating: $[P_A(p)]^N$**	**Quantity-rating: $-\log_2(P_Q(p))$**	**QQ-rating**
0.5	0.25	*...in Moscow*	.0625	2	.125
0.5	0.25	*...in Petersburg*	.0625	2	.125
0.25	0.125	*...on Tallinsk. in M.*	.0039	3	.0117
1	0.5	*...in Russia*	1	1	1
1	1	*...somewhere*	1	0	0

There was some concern that this worked out well because $N = 4$ happened to be selected. So: the same chart for $N = 1$:

A	**Q**	**Proposition**	**Quality**	**Quantity**	**QQ**
0.5	0.25	*...in Moscow*	0.5	2	1
0.5	0.25	*...in Petersburg*	0.5	2	1
0.25	0.125	*...on Tallinsk. in M.*	0.25	3	0.75
1	0.5	*...in Russia*	1	1	1
1	1	*...somewhere*	1	0	0

...which doesn't work as well, which is why one chooses N to be greater than 1. For $N = 10$:

A	**Q**	**Proposition**	**Quality**	**Quantity**	**QQ**
0.5	0.25	*...in Moscow*	0.001	2	0.002
0.5	0.25	*...in Petersburg*	0.001	2	0.002
0.25	0.125	*...on Tallinsk. in M.*	0	3	0
1	0.5	*...in Russia*	1	1	1
1	1	*...somewhere*	1	0	0

In other words, for higher values of N, we get the same results, only skewed even further (in the right direction).

1.2. 10 countries, 10 cities per country, 10 streets per city

What happens if the number of possibilities increases? Suppose there are ten countries, ten cities per country, ten streets per city. Then:

A	Q	Proposition	Quality	Quantity	QQ
0.1	0.01	*...in Moscow*	0.1	6.6439	0.6644
0.1	0.01	*...in Petersburg*	0.1	6.6439	0.6644
0.01	0.001	*...on Tallinsk. in M.*	0.01	9.9658	0.0997
1	0.1	*...in Russia*	1	3.3219	3.3219
1	1	*...somewhere*	1	0	0

$N = 1$

A	Q	Proposition	Quality	Quantity	QQ
0.1	0.01	*...in Moscow*	0.0001	6.6439	0.0007
0.1	0.01	*...in Petersburg*	0.0001	6.6439	0.0007
0.01	0.001	*...on Tallinsk. in M.*	0	9.9658	0
1	0.1	*...in Russia*	1	3.3219	3.3219
1	1	*...somewhere*	1	0	0

$N = 4$

A	Q	Proposition	Quality	Quantity	QQ
0.1	0.01	*...in Moscow*	0	6.6439	0
0.1	0.01	*...in Petersburg*	0	6.6439	0
0.01	0.001	*...on Tallinsk. in M.*	0	9.9658	0
1	0.1	*...in Russia*	1	3.3219	3.3219
1	1	*...somewhere*	1	0	0

$N = 10$

So: as we introduce more possible cities or countries, the results once again fall even more sharply in favor of Potts's results.

2. VARYING THE ANSWERER'S CERTAINTY

The above scenarios assumed that, as far as the answerer is concerned, Moscow is as likely as any other city in Russia. What if that's not true? What if the answerer knows that Barbara lives in Russia but, out of ten possible cities in Russia, he's relatively certain that it's Moscow that Barbara lives in?

Suppose that there are ten countries, with ten cities per country (we won't have to go to the street level for this scenario). For the answerer, the QQ-rating of the answer *She lives in Russia* is going to be 3.3219 regardless—remember that Barbara lives in Russia in 100% of the answerer's belief worlds and in 10% of the questioner's:

Quality: $[P_A(\textit{She lives in Russia})]^N = 1^N = 1$
Quantity: $-\log_2(P_Q(\textit{She lives in Russia})) = -\log_2(0.1) = 3.3219$

Note, too, that the Quantity-rating of *She lives in Moscow* will always be the same, since we're still presuming that the questioner has no idea which of the ten cities Barbara lives in and thus gives it the same 1/100 chance as any other city:

Quantity: $-\log_2(P_Q(\textit{She lives in Moscow})) = -\log_2(0.01) = 6.6439$

Now we have a fairly easy-to-calculate formula: the answerer will say *She lives in Moscow* instead of *She lives in Russia* if the QQ-rating of the former is higher than that of the latter, i.e., greater than 3.3219. First, a quick chart of some possibilities for various values of *N*, given a percentage certainty of her living in Moscow on the part of the speaker:

% Certainty	***N* = 1**	***N* = 2**	***N* = 3**	***N* = 4**	***N* = 5**	***N* = 10**	***N* = 20**
70	***4.651***	3.255	2.279	1.595	1.12	0.188	0.005
72.5	***4.817***	***3.492***	2.532	1.836	1.33	0.267	0.011
75	***4.983***	***3.737***	2.803	2.102	1.58	0.374	0.021
77.5	***5.149***	***3.99***	3.093	2.397	1.86	0.519	0.041
80	***5.315***	***4.252***	***3.402***	2.721	2.18	0.713	0.077
82.5	***5.481***	***4.522***	***3.731***	3.078	2.54	0.97	0.142
85	***5.647***	***4.8***	***4.08***	***3.468***	2.95	1.308	0.258
87.5	***5.813***	***5.087***	***4.451***	***3.895***	***3.41***	1.748	0.46
90	***5.979***	***5.382***	***4.843***	***4.359***	***3.92***	2.317	0.808
92.5	***6.146***	***5.685***	***5.258***	***4.864***	***4.5***	3.047	1.397
95	***6.312***	***5.996***	***5.696***	***5.411***	***5.14***	***3.978***	2.382
97.5	***6.478***	***6.316***	***6.158***	***6.004***	***5.85***	***5.158***	***4.004***
100	***6.644***	***6.644***	***6.644***	***6.644***	***6.64***	***6.644***	***6.644***

The bold-italic numbers are the ones higher than 3.3219. In other words, if we'd hypothesized *N* to be 1, you wouldn't have to be very certain at all that she lived in Moscow; for *N* = 4, you need to be perhaps 83% certain; for *N* = 10, it jumps to 93% certainty.

That's not a great result for Potts, perhaps. Is 83% certainty enough to warrant saying (the more informative) S*he lives in Moscow* instead of (the more true) *She lives in Russia*? On the other hand, once you introduce more cities and countries—say, 100 countries, with twenty cities per country—the numbers are:

Quality: $[P_A(\textit{She lives in Russia})]^N = 1^N = 1$
Quantity: $-\log_2(P_Q(\textit{She lives in Russia})) = -\log_2(0.01) = 6.6439$

Quantity: $-\log_2(P_Q(\textit{She lives in Moscow})) = -\log_2(0.0005) = 10.966$

% Certainty	*N* = 1	*N* = 2	*N* = 3	*N* = 4	*N* = 5	*N* = 10	*N* = 20
70	***7.676***	5.373	3.761	2.633	1.84	0.31	0.009
72.5	***7.95***	5.764	4.179	3.03	2.2	0.44	0.018
75	***8.224***	6.168	4.626	3.47	2.6	0.618	0.035
77.5	***8.498***	6.586	5.104	3.956	3.07	0.857	0.067
80	***8.773***	***7.018***	5.614	4.492	3.59	1.177	0.126
82.5	***9.047***	***7.464***	6.157	5.08	4.19	1.602	0.234
85	***9.321***	***7.923***	***6.734***	5.724	4.87	2.159	0.425
87.5	***9.595***	***8.396***	***7.346***	6.428	5.62	2.885	0.759
90	***9.869***	***8.882***	***7.994***	***7.195***	6.48	3.824	1.333
92.5	***10.143***	***9.383***	***8.679***	***8.028***	***7.43***	5.029	2.306
95	***10.417***	***9.897***	***9.402***	***8.932***	***8.49***	6.566	3.931
97.5	***10.692***	***10.424***	***10.16***	***9.91***	***9.66***	***8.513***	6.609
100	***10.97***	***10.97***	***10.97***	***10.97***	***10.97***	***10.97***	***10.97***

The numbers look at least a little better.

In fact, we can work out the exact numbers. For *k* nations with *x* cities each, and certainty *c*:

Quality: $[\mathrm{P_A}(\textit{She lives in Russia})]^N = 1^N = 1$

Quantity: $-\log_2(\mathrm{P_Q}(\textit{She lives in Russia})) = -\log_2\left(\frac{1}{k}\right)$

Quality: $[\mathrm{P_A}(\textit{She lives in Moscow})]^N = c^N$

Quantity: $-\log_2(\mathrm{P_Q}(\textit{She lives in Moscow})) = -\log_2\left(\frac{1}{kx}\right)$

She lives in Moscow is the preferred assertion when $\mathrm{QQ_{Moscow}} > \mathrm{QQ_{Russia}}$. Solving for *c*:

$$c^N \cdot -\log_2\left(\frac{1}{kx}\right) > 1 \cdot -\log_2\left(\frac{1}{k}\right) \equiv c^N > \log_2\left(\frac{1}{k}\right) / \log_2\left(\frac{1}{kx}\right) \equiv c^N > \frac{\log_2(k)}{\log_2(kx)} \equiv$$

$$c > \sqrt[N]{\frac{\log(k)}{\log(kx)}} = \sqrt[N]{\frac{\log(k)}{\log(k)+\log(x)}}$$

This last formula is fairly easy to comprehend. Certainly, the larger the *N*, the larger *c* will need to be. If we've picked a particular *N* (e.g., 4), then increasing the number of countries *k* will increase *c*, but increasing the number of cities *x* will **decrease** *c*.[1] Additionally, the need for certainty depends ultimately on the *relative* sizes of *k* and *x* rather than their absolute values—the

[1] Why? Intuitively: because the more countries there are under consideration, the more helpful it is to know that it's Russia as opposed to some other country, in which case knowing Moscow as opposed to Petersburg doesn't make as much difference. But the more cities there are, the more helpful it is to know which particular city.

Also note that if $x = 1$, then $\log(x) = 0$, so the fraction under the radical = 1, so the overall formula reduces to 1—that is to say, if there's only one city under consideration in each country (e.g., we know Barbara lives in the capital), naming the city is equivalent to naming the country. Conversely, if $k = 1$, then $\log(k) = 0$ and the overall formula reduces to 0—that is to say, if there's only one country under consideration, naming the country will always be less informative.

certainty needed to name a city in the “two country/two cities each” case is exactly the same as in the “30 countries/30 cities each” case; the certainty needed to name a city in the “2 country/4 cities each” case is exactly the case as in the “3 country/9 cities each”, “4 country/16 cities each”, “50 countries/2500 cities each” cases.

So we can work out, for various numbers of countries and cities, how certain the speaker needs to be about the more specific sentence in order to say it:

Countries *k*	**Cities per country *x***	***N***	**% certainty**
2	2	4	84.09
4	2	4	75.98
8	2	4	70.71
16	2	4	66.87
100	2	4	60.14
2	2	4	84.09
2	4	4	90.36
2	8	4	93.06
2	16	4	94.57
2	100	4	96.56
2	2	4	84.09
2	2	5	87.06
2	2	10	93.3
2	2	20	96.59

Whether these numbers look good for Potts’s theory of integrated pragmatic values depends, I suppose, on how well they align with our intuitions of how certain one has to be about a more specific proposition before uttering it.

3. HOMEWORK – DUE BY FRIDAY, MARCH 6, 5:00 PM

3.1. Not the question

Consider the following games of chance. (This illustration is not original to me, but alas, I’ve long forgotten its source.)

> **Game A:** The game costs $10 to play. You roll three dice. If they come up triple sixes, you receive nothing. Anything else, you receive $100 (plus your original $10).
>
> **Game B:** The game costs $10 to play. You roll three dice. If they come up triple sixes, you receive $100 (plus your original $10). Anything else, you receive nothing.

It doesn't take much mathematical knowledge to recognize the first game as a good idea and the second game as a bad idea.[2]

Now consider the following two scenarios:

> **Scenario 1:** You're standing outside a pharmacy with $10 in your pocket. You have three minutes to buy the medication that will save your life, which costs $10. As you're about to open the door, a man nearby offers to play Game A with you.
>
> **Scenario 2:** You're standing outside a pharmacy with $10 in your pocket. You have three minutes to buy the medication that will save your life, which costs $100. As you're about to open the door, a man nearby offers to play Game B with you.

Would you play the game in Scenario A? Would you play it in Scenario B?

That's not the homework question. That's something to consider while answering the following.

3.2. The question

In Potts's original scenario (bolded emphasis added):

> The players' interpretation of this question—in particular, the level of detail requested by *where*—will be heavily conditioned by a range of contextual factors, as will player *A*'s reply. Let's assume that **the players' shared goal is to buy plane tickets**, and thus that (2) is interpreted by them as equivalent to the question of which *city* Barbara lives in.

Suppose you know she lives in Russia but have doubts about which city. How certain *do* you have to be to answer "Moscow"? Seventy-five percent certain? Eighty? Ninety? Ninety-nine? (Bearing in mind that it's hard to be 100% certain of anything—you may have talked to Barbara yesterday about how much she likes living in Moscow, and in the last 24 hours her company was reorganized and she's being relocated to Paris.)

Suppose it's clear that the questioner is not buying plane tickets, but rather is looking for a resident of Moscow she can email, to ask them stop by Lenin's Tomb and double-check an inscription she's writing a paper about. In that case, how certain do you have to be to answer "Moscow"?

And ultimately, the question is this: if there's a difference in the above two situations, then

- why is that, and more importantly
- how does Gricean reasoning explain the difference, if at all, and
- how does Potts's analysis explain the difference, if at all?

Answer in essay form (i.e., not anything *long*, but in complete sentences and paragraphs).

[2] Mathematically: the expected payout for the first game is $(-10 \cdot 1/216) + (100 \cdot 215/216) \approx 99.49$—that is, if you play enough, you'll average a gain of $99.49 for each game you play—and the expected payout for the second game is $(-10 \cdot 215/216) + (100 \cdot 1/216) = -9.49$—if you play enough, you'll lose an average of $9.49 per game.