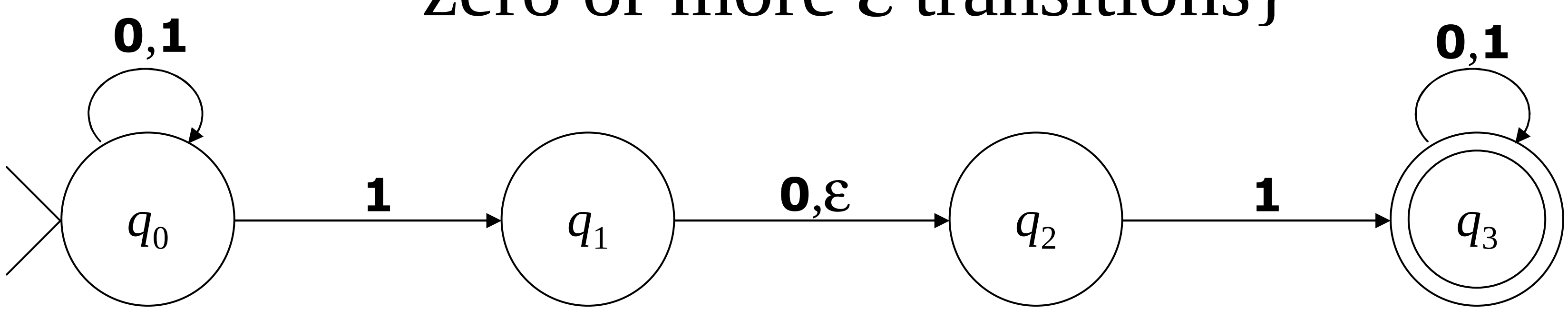
NFA examples

LING 106
Oct. 20, 2008

Proof, Part II, Continued

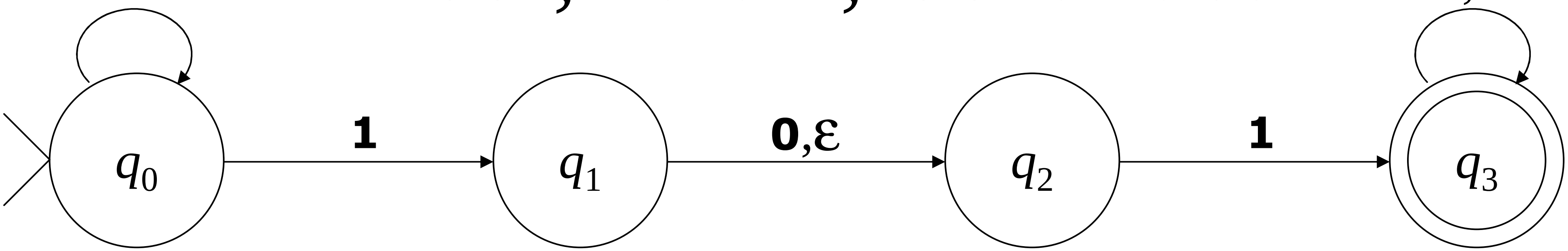
- For any $R \in Q_D$, define $E(R)$ to be the set of states that can be reached from R by going along empty-string transitions, including the states in R themselves. i.e., for $R \subseteq Q_N$,

$$E(R) = \{q \mid q \text{ can be reached from } R \text{ by following zero or more } \varepsilon \text{ transitions}\}$$



- For example, in $\boldsymbol{N_1}$:
  - $E(\{q_1\}) = \{q_1, q_2\}$
  - $E(\{q_0\}) = \{q_0\}$
  - $E(\{q_0, q_1\}) = \{q_0, q_1, q_2\}$

Proof, Part II, Continued



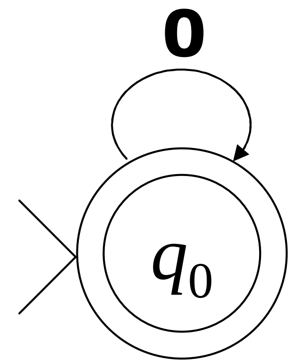
- Then:

$$\delta_D(<R, x>) = \{q \in Q_N \mid q \in E(\delta_N(<r, x>)) \text{ for some } r \in R\}$$

- For instance:

$$\begin{aligned}
&\delta_D(<\{q_0\}, \mathbf{1}>) = \\
&\quad \{q \in Q_N \mid q \in E(\delta_N(<r, x>)) \text{ for some } r \in R\} = \\
&\quad \{q \in Q_N \mid q \in E(\delta_N(<q_0, \mathbf{1}>))\} = \\
&\quad \{q \in Q_N \mid q \in E(\{q_0, q_1\})\} = \\
&\quad \{q \in Q_N \mid q \in \{q_0, q_1, q_2\})\} = \\
&\quad \{q_0, q_1, q_2\}
\end{aligned}$$

Perhaps an Example?



- How do we turn this machine into a DFA?

$\Sigma_D = \Sigma_N = \{\mathbf{0}, \mathbf{1}\}$

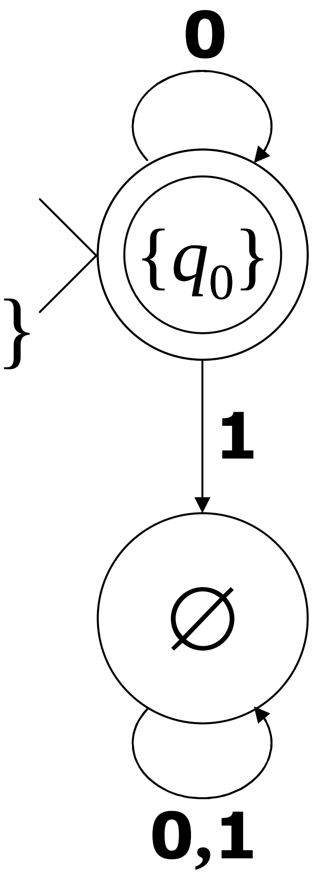
$Q_D = \wp(Q_N) = \{\varnothing, \{q_0\}\}$

$s_D = E(\{s_N\}) = \{q_0\}$

$F_D = \{R \in Q_D \mid R \text{ contains an accept state of } N\} = \{\{q_0\}\}$

$\delta_D(<R, x>) = \{q \in Q_N \mid q \in E(\delta_N(<r, x>)) \text{ for some } r \in R\}$

	0	**1**
$\{q_0\}$	$\{q_0\}$	$\varnothing$
$\varnothing$	$\varnothing$	$\varnothing$

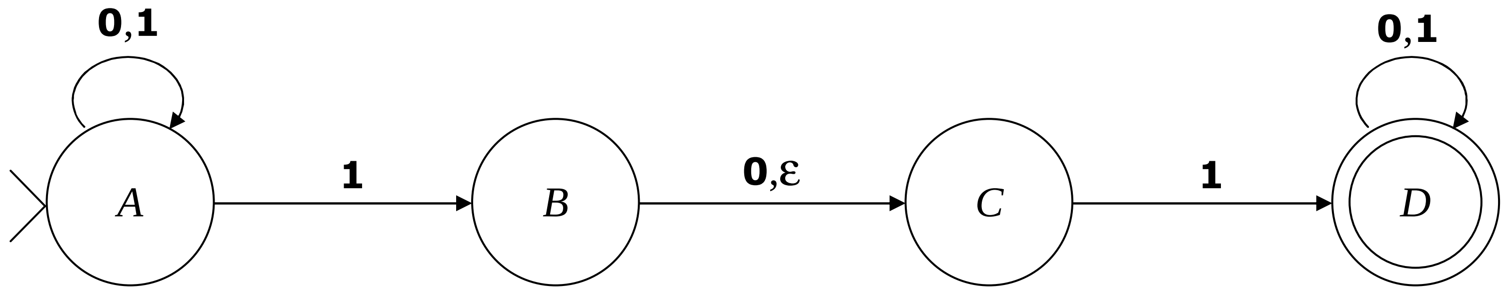


$Q_N = \{A, B, C, D\}$

$\Sigma_N = \{\mathbf{0},\mathbf{1}\}$

$s_N = A$

$F_N = \{D\}$



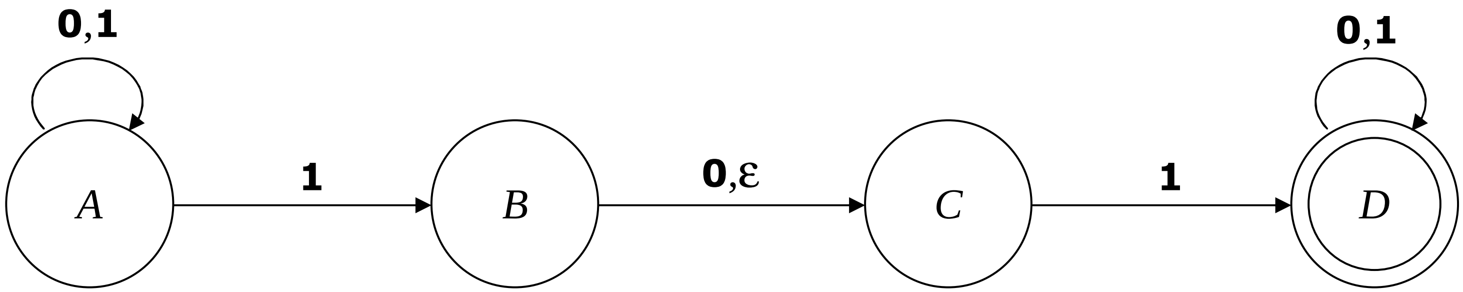
	0	**1**	ε
A	$\{A\}$	$\{A, B\}$	$\varnothing$
B	$\{C\}$	$\varnothing$	$\{C\}$
C	$\varnothing$	$\{D\}$	$\varnothing$
D	$\{D\}$	$\{D\}$	$\varnothing$

- $\Sigma_D = \Sigma_N = \{\mathbf{0},\mathbf{1}\}$
- $Q_D = \wp(Q_N) =$

$\{\varnothing$, A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, ACD, BCD, ABCD$\}$

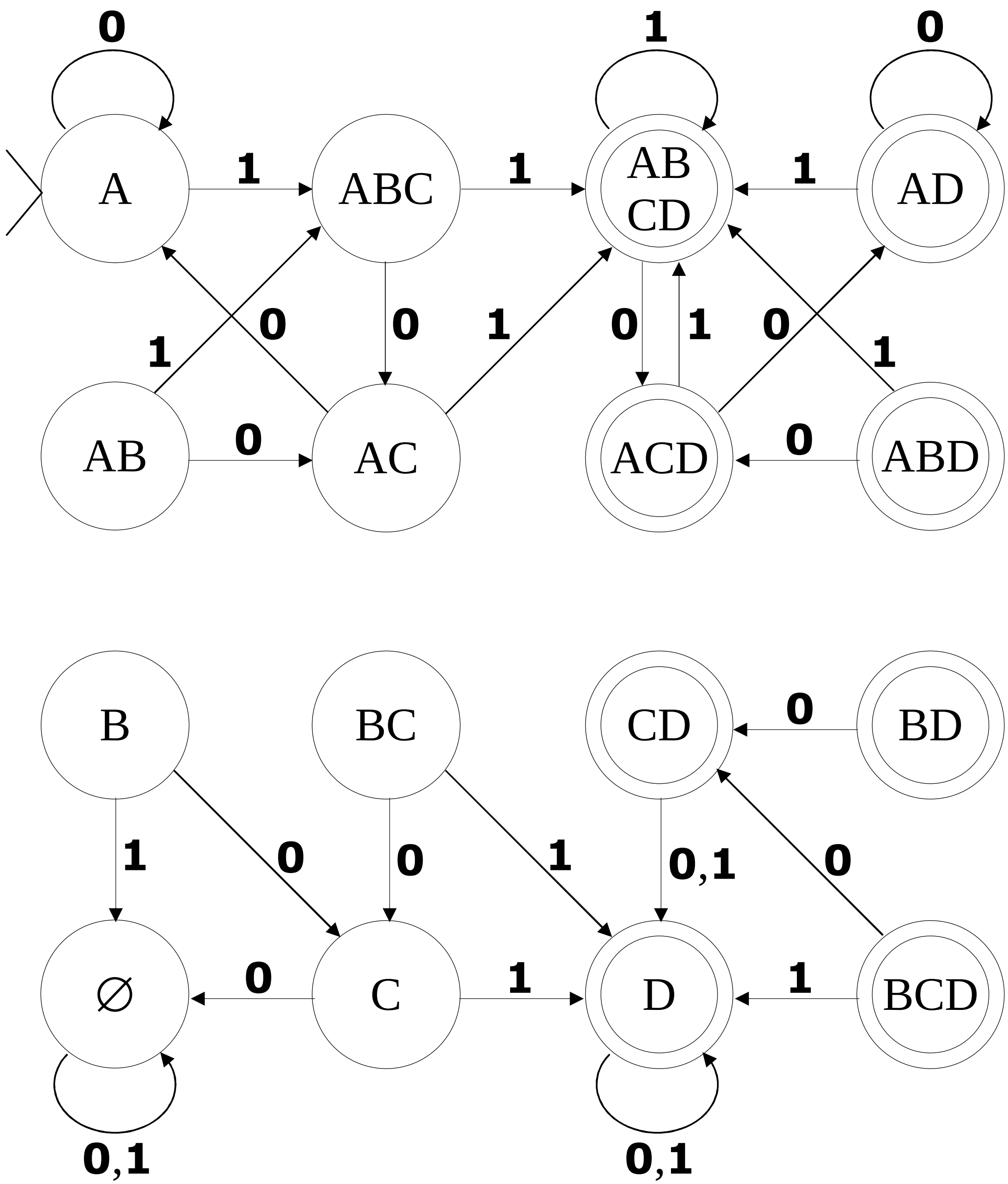
- $s_D = E(\{s_N\}) = E(\{A\}) =$ A
- $F_D = \{R \in Q_D \mid R \text{ contains an accept state of } \mathbf{N}\} =$

{D, AD, BD, CD, ABD, ACD, BCD, ABCD}

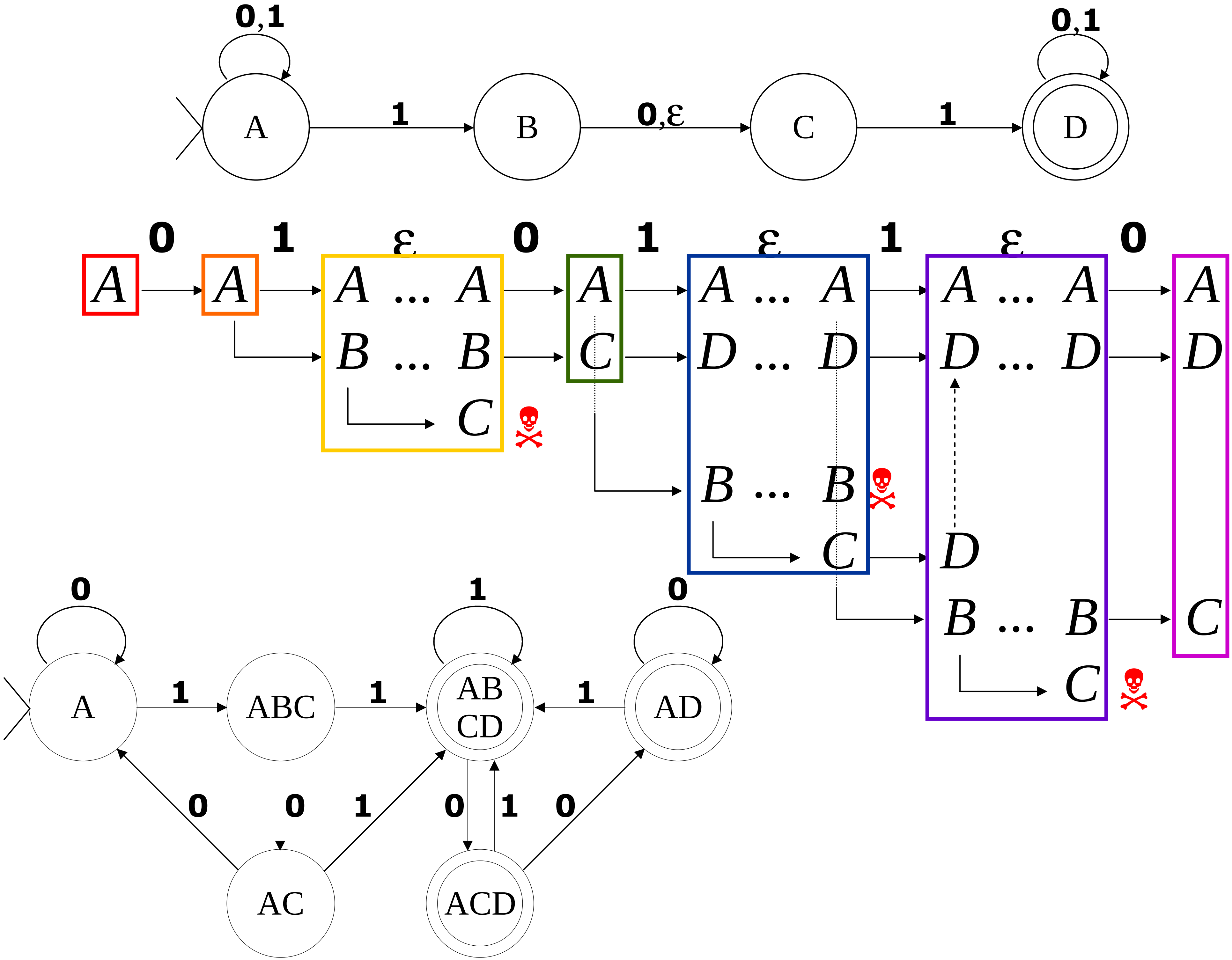


	0	1	ε
A	$\{A\}$	$\{A, B\}$	$\varnothing$
B	$\{C\}$	$\varnothing$	$\{C\}$
C	$\varnothing$	$\{D\}$	$\varnothing$
D	$\{D\}$	$\{D\}$	$\varnothing$

	0	1
∅	∅	∅
A	A	ABC
B	C	∅
C	∅	D
D	D	D
AB	AC	ABC
AC	A	ABCD
AD	AD	ABCD
BC	C	D
BD	CD	D
CD	D	D
ABC	AC	ABCD
ABD	ACD	ABCD
ACD	AD	ABCD
BCD	CD	D
ABCD	ACD	ABCD



	0	1
∅	∅	∅
A	A	ABC
B	C	∅
C	∅	D
D	D	D
AB	AC	ABC
AC	A	ABCD
AD	AD	ABCD
BC	C	D
BD	CD	D
CD	D	D
ABC	AC	ABCD
ABD	ACD	ABCD
ACD	AD	ABCD
BCD	CD	D
ABCD	ACD	ABCD



0,1
A
1
B
0,ε
C
1
D
0,1
0 1 ε 0 1 ε 1 ε 0
A A A … A A A … A A … A A
B … B C D … D D … D D
C
B … B
C D
B … B C
C
0 1 0
A 1 ABC 1 AB CD 1 AD
0 0 1 0 1 0
AC ACD