

## LING 106: Midterm

Begins: February 25, 2009, 8:30pm

Ends: March 4, 2009, 8:30pm

This is the midterm exam. Please read through *all* of the instructions on this page before proceeding. Failure to follow *any* instruction on this page will cost you **five points**. In part, because this is a formal course, and following precise directions is part of the material. And in part because these instructions aren't arbitrary; they're here to make the exam easier to grade, which means that, if you don't follow them, you've made my life harder. Don't annoy the guy in charge of your grade.

- The exam itself consists of 8 pages plus this page of instructions (“page 0”) and an appendix. Do not miss any pages.
- The exam is due at **8:30pm on March 4th**. No exceptions without a note from a dean, doctor, or clergyman. If you turn it in late, you will lose **ten points every twelve hours**, rounded up.
- Your answers should appear in the same order as the questions on the exam, and each problem (not each question!) should begin on a new page. You can achieve this by either...
  - Printing out the exam, writing your answers in the space provided, and turning in the exam pages. Use the back of the page if necessary, or attach another sheet of paper immediately after the page; or
  - Using other paper (typed, printed, handwritten on blank paper, lined paper, etc.), starting a new page where appropriate.

Either way, indicate at the top of each page which question is being answered. *Do not submit page 0 or the appendix as part of your answers*. If you do not answer the bonus question, you do not need to include that page.

- Your name should appear on each page of your answers.

You may use your notes and any notes from the class. You **may not** discuss the exam with classmates, or indeed with anyone else other than myself.

Good luck!

**1. WARMUP: TRUE/FALSE QUESTIONS****25 points****Question 1:** Identify the statements in (a)-(j) as either true or false. **[1 pt each]**Given the set  $Q = \{0, 1\}$ :

- |    |                     |    |                              |
|----|---------------------|----|------------------------------|
| a. | $0 \in Q$           | f. | $\emptyset \subseteq Q$      |
| b. | $\{0\} \in Q$       | g. | $\{0\} \in \wp(Q)$           |
| c. | $0 \subseteq Q$     | h. | $\{0\} \subseteq \wp(Q)$     |
| d. | $\{0\} \subseteq Q$ | i. | $\emptyset \in \wp(Q)$       |
| e. | $\emptyset \in Q$   | j. | $\emptyset \subseteq \wp(Q)$ |

**Question 2:** Identify the statements as true or false, and explain your answer. **[5 pts each]**

- a.  $g^{-1}$  is a function, where  
 $g : \mathbf{L} \rightarrow \mathbf{L}$ , with  $\mathbf{L}$  = the set of letters in English, and  
 $g(\langle x, y \rangle)$  iff  $y$  is the letter following the first occurrence of  $x$  in the sentence  
**sympathizing would fix quaker objectives.**<sup>1</sup>
- b. Given the two sentences of Cebuano at the bottom of the page, we can hypothesize that the two underlined words (*kuhaa* and *anaa*) have the same syntactic category. **[5 pts]**
- c. Given the two sentences of Cebuano at the bottom of the page, we can hypothesize that the two bolded words (*pagtoo* and *bana*) have the same syntactic category. **[5 pts]**
- (i) Pinangga, kuhaa ang imong maskará; patan-awa akó pagbalik sa imong kaanyag; patan-awa akó kay buut ko sublion ang akong **pagtoo** nga si ibabaw ning tanáng kahayag, sa ibabaw ning tanáng kalipay, nagahari sa gihapon ang imong katahum; pahagka akó....
- (ii) Si Torcuata igkita ta kanunay sa mga sine. “Formal” kaayo nga maglingkod; magpakapal sa pinulpog. Ang nawong daw bantiyong ahat. Sa iyang tupad, anaa ang iyang “bana”. Pamatia ninyo siyá kong adunay higalang igkita: “Ipailá kó kanimo ang akong **bana** nga si Teótimo Lamisa.”

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<sup>1</sup> For example,  $\langle z, i \rangle \in g$ , because **i** follows the first (and only) occurrence of **z** in the sentence.  $\langle e, c \rangle \notin g$ , because **r** follows the first occurrence of **e**.

**2. SET THEORY, PART ONE****10 points**

Let set  $S$  = the set of the fifty states in the U.S. Set  $A$  is defined as follows:

- Illinois  $\in A$ ;
- If  $x \in A$ ,  $y \in S$ , and the governor of  $y$  was born in  $x$ , then  $y \in A$ ;
- If  $x \in A$ ,  $y \in S$ , and the governors of  $x$  and  $y$  have the same first name, then  $y \in A$ ;
- Nothing else is in  $A$ .

For the purposes of this question, the governors of the fifty states and their birthplaces are the ones given in Appendix A, at the end of the exam. If any of the information is inaccurate, use it anyways and ignore reality. Additionally, the “first name” of a governor is the name listed there—so, e.g., Bob Riley and Bobby Jindal do not have the same first name.

As a helpful tip:  $7 < |A| < 15$ .

**Question:** Express  $A$  in list notation. (You may wish to show your work.)

**3. SET THEORY, PART TWO****10 points**

Consider the set of strings  $B$  over the alphabet  $\Sigma = \{\mathbf{n}, \mathbf{a}\}$ , organized here alphabetically by length:

$$B = \{ \varepsilon, \mathbf{a}, \mathbf{n}, \mathbf{aa}, \mathbf{nn}, \mathbf{aaa}, \mathbf{ana}, \mathbf{nan}, \mathbf{nnn}, \mathbf{aaaa}, \mathbf{anna}, \mathbf{naan}, \mathbf{nnnn}, \mathbf{aaaaa}, \mathbf{aanaa}, \mathbf{anana}, \mathbf{annna}, \mathbf{naaan}, \mathbf{nanan}, \mathbf{nnann}, \mathbf{nnnnn}, \dots \}$$

Note: if the pattern isn't sufficiently clear, you can ask me for clarification. Note that asking me to describe the pattern won't get you very far, since that's what Questions 1 and 2 ask you to do. But I can answer at least some questions concerning what is or isn't in  $B$ .

**Question 1:** Express  $B$  in predicate notation. **[5 pts]**

**Question 2:** Express  $B$  in recursive notation. **[5 pts]**

**4. SET OPERATIONS****16 points**Given the universe  $U$ :

$$U = \{ \text{Morticia, Gomez, Fester, Wednesday, Pugsley} \}$$

The facts about these people vary somewhat from comic to TV show to movie. For the purposes of this question: Morticia and Gomez are married, Wednesday is their daughter, Pugsley is their son, and Fester and Gomez are brothers. Note, too, that the entire universe for the purposes of this question is these five people (you can safely ignore Grandmama, Itt, Thing, Lurch, and any other members of their family or household).

In your answers, feel free to use initials instead of writing out the full names; after all, as you well know, the contents of a set don't depend on how you refer to them.

**Question:** Express each of the following in *list notation*.

a.  $\{x \mid x \text{ has a brother}\}$  **[3 pts]**

b.  $\{x \mid x \text{ is more than eight letters long}\}$  **[3 pts]**

c.  $\wp(F)$ , where  
 $F = ((C \cup D) - (C \cap D))$   
 $C = \{y \mid y \text{ is male}\}$   
 $D = \{y \mid y \text{ has a child}\}$  **[5 pts]**

d.  $C \times C'$ , where  $C$  is defined as in (c), above. **[5 pts]**

**5. FUNCTIONS AND DETERMINISTIC FINITE STATE AUTOMATA****14 points**

A deterministic finite state automaton is a 5-tuple  $\langle Q, \Sigma, \delta, s, F \rangle$ , with  $\delta: Q \times \Sigma \rightarrow Q$ . Suppose that we have a DFA with at least two symbols in the alphabet.

**Question 1:** In terms of one-to-one vs. many-to-one, there are three options:

- $\delta$  *must* be one-to-one and *cannot* be many-to-one.
- $\delta$  *cannot* be one-to-one and *must* be many-to-one.
- $\delta$  could be either one-to-one or many-to-one.

Which of these statements is true? Explain your answer. **[7 pts]**

**Question 2:** In terms of onto vs. into, there are three options:

- $\delta$  *must* be onto and *cannot* be into.
- $\delta$  *cannot* be onto and *must* be into.
- $\delta$  could be either onto or into.

Which of these statements is true? Explain your answer. **[7 pts]**

Warning: this question will take some thought! You might want to experiment with a few different finite state automata, via diagram or 5-tuple. Also: your answer, while it may include functions or diagrams for reference, must be written in paragraph form, in complete sentences. (Part of “formal reasoning” is the ability to present that reasoning!)

**6. READING A DETERMINISTIC FSA**

**15 points**

Consider the finite-state automaton below.

**Question 1:** Provide the specifications for the FSA, i.e. its 5-tuple. **[10 pts]**

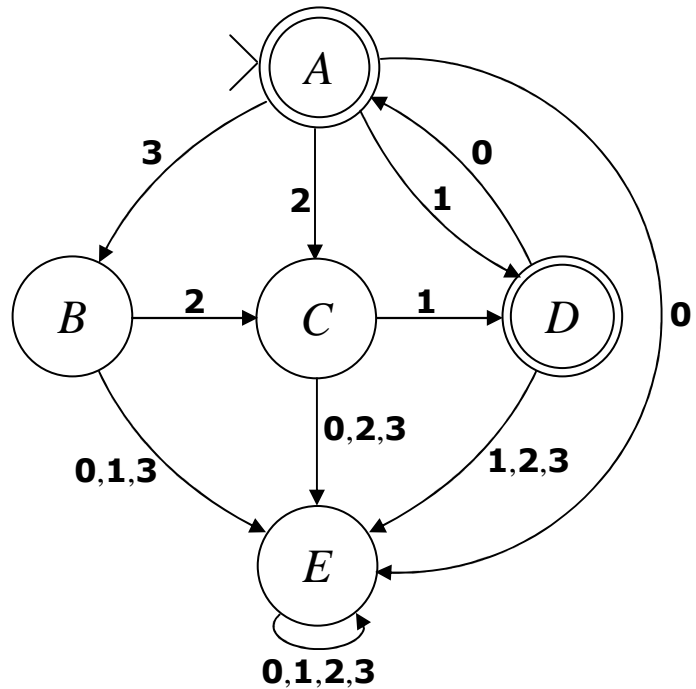
$Q =$

$\Sigma =$

$s =$

$F =$

$\delta =$



**Question 2:** Let  $K = \{s \mid \mathbf{3}$  is a substring of  $s$  and  $|s| \leq 6\}$ . For example, **3**, **233**, and **031212** are all elements of  $K$ , while none of  $\epsilon$ , **20**, or **3101010** are elements of  $K$ . Provide a list of all strings in  $K$  accepted by the FSA above. (For reference, considering only strings over  $\Sigma$ ,  $|K| = 4,368$  if I did the math right; but regardless, you'll be writing down fewer than ten strings.) **[5 pts]**

**7. DESIGNING A DETERMINISTIC FSA****10 points**

**Question:** For the alphabet  $\{\mathbf{b}, \mathbf{o}, \mathbf{p}\}$ , prove that language  $L$  is regular by providing the specifications for a deterministic finite state automaton that models it:

$$L = \{s \mid \text{any occurrence of } \mathbf{b} \text{ in } s \text{ is followed } \underline{\text{at some point}} \text{ by an occurrence of } \mathbf{o}\}$$

e.g., **bop**, **obbpo**, **pp**, and **boopboop** are all in  $L$ ; **b**, **bob**, **obpopbp** are all not. (Once again, you can email me with questions about what is or isn't in the language.)

**You do not need to draw the machine;** you may if you like, but the grading will depend solely on your specification of the 5-tuple. Specify  $\delta$  in whatever fashion you find easiest (but be sure that your specification is clear and complete). Use as few states as possible.

 $Q =$ 
 $\delta =$ 
 $\Sigma =$ 
 $s =$ 
 $F =$

**8. BONUS QUESTION: DECIPHERING STRINGS 5 bonus points**

**PLEASE NOTE!** This is strictly a bonus question. It is not required. It cannot raise your score over 100%. And it *cannot be used to make up for skipped questions*, only for points lost on questions you genuinely attempted.

Given the following transformations:

$C$  = Caesar shift each letter forward by 4.

Example:  $C(\mathbf{example}) = \mathbf{ibeqtpi}$

$P$  = Replace each letter with the one that follows it in the pangram

cwm fjord bank glyphs vext quiz  
replacing z with c if necessary.<sup>2</sup>

Example:  $P(\mathbf{example}) = \mathbf{xtnfhyx}$

$R$  = Reverse each set of five letters.

Example:  $R(\mathbf{the ex|ample}) = \mathbf{xee ht|elpma}$

$Q$  = [a mystery transformation to be determined]

**Question:** Find  $x$ , such that:

$$C^{-1}(P^{-1}(R(Q(C(P(x)))))) = \mathbf{lflv rkcrf fqfrs foxqn wkftvq}$$

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<sup>2</sup> A pangram is something that contains each letter of the alphabet at least once. This one means “The symbols carved on the sides of a narrow inlet in the steep-walled basin on a mountain confused an eccentric person.”

## APPENDIX A: THE FIFTY STATES AND THEIR GOVERNORS

State	Governor	Born in...
Alabama	Bob Riley	Alabama
Alaska	Sarah Palin	Idaho
Arizona	Jan Brewer	California
Arkansas	Mike Beebe	Arkansas
California	Arnold Schwarzenegger	Austria
Colorado	Bill Ritter	Colorado
Connecticut	M. Jodi Rell	Virginia
Delaware	Jack Markell	Delaware
Florida	Charlie Crist	Pennsylvania
Georgia	Sonny Perdue	Georgia
Hawaii	Linda Lingle	Missouri
Idaho	Butch Otter	Idaho
Illinois	Pat Quinn	Illinois
Indiana	Mitch Daniels	Pennsylvania
Iowa	Chet Culver	D.C.
Kansas	Kathleen Sebelius	Ohio
Kentucky	Steve Beshear	Kentucky
Louisiana	Bobby Jindal	Louisiana
Maine	John Baldacci	Maine
Maryland	Martin O'Malley	D.C.
Massachusetts	Deval Patrick	Illinois
Michigan	Jennifer Granholm	Canada
Minnesota	Tim Pawlenty	Minnesota
Mississippi	Haley Barbour	Mississippi
Missouri	Jay Nixon	Missouri

State	Governor	Born in...
Montana	Brian Schweitzer	Montana
Nebraska	Dave Heineman	Nebraska
Nevada	Jim Gibbons	Nevada
New Hampshire	John Lynch	Massachusetts
New Jersey	Jon Corzine	Illinois
New Mexico	Bill Richardson	California
New York	David Paterson	New York
North Carolina	Beverly Perdue	Virginia
North Dakota	John Hoeven	North Dakota
Ohio	Ted Strickland	Ohio
Oklahoma	Brad Henry	Oklahoma
Oregon	Ted Kulongoski	Missouri
Pennsylvania	Ed Rendell	New York
Rhode Island	Donald Carcieri	Rhode Island
South Carolina	Mark Sanford	Florida
South Dakota	Mike Rounds	South Dakota
Tennessee	Phil Bredesen	New Jersey
Texas	Rick Perry	Texas
Utah	Jon Huntsman, Jr.	California
Vermont	Jim Douglas	Massachusetts
Virginia	Tim Kaine	Minnesota
Washington	Christine Gregoire	Washington
West Virginia	Joe Manchin	West Virginia
Wisconsin	Jim Doyle	D.C.
Wyoming	Dave Freudenthal	Wyoming