LING 106: Homework 6

Due: Monday, April 20, 2009 (5:00 pm)

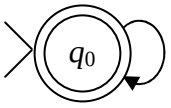
1. READING FINITE STATE TRANSDUCERS

Let's consider a few FSTs that operate on strings of written text. For these purposes:

$$\Sigma = \{\textbf{a}, \textbf{b}, \textbf{c}, \textbf{d}, \textbf{e}, \textbf{f}, \textbf{g}, \textbf{h}, \textbf{i}, \textbf{j}, \textbf{k}, \textbf{l}, \textbf{m}, \textbf{n}, \textbf{o}, \textbf{p}, \textbf{q}, \textbf{r}, \textbf{s}, \textbf{t}, \textbf{u}, \textbf{v}, \textbf{w}, \textbf{x}, \textbf{y}, \textbf{z}, \textbf{\#}\}$$

where the special symbol **#** marks the end of a word. (It's easier to read than a space.) Spaces can be ignored, or added for ease of reading; **hi#there#** and **hi # there #** are the same string. (So is **h i#th er e#**, but don't write that.)

For example, F_1:

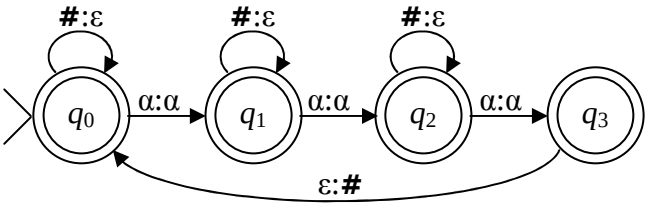


a:**d** ... **y**:**b**
b:**e** **w**:**z** **z**:**c**
c:**f** **x**:**a** **#**:**#**

Machine F_1

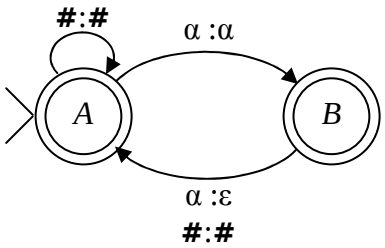
Note that there are 27 input/output pairs that label the transition, of which 8 are shown. You can infer the rest.

Question 1: What does F_1 do? That is, state its operation in words (e.g., the operations of machines we saw in class could be described as "deleting the last symbol of a string" or "swapping the **0**s and **1**s"). What is the output that the machine associates with the input **meet # me # at # the # library #**?

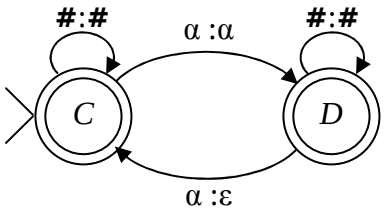


Machine F_2

Question 2: What does F_2 do? (Remember that α means "a symbol of the alphabet not otherwise appearing in the machine".) Once again, if the input is **meet # me # at # the # library #**, what is the output?



Machine F_3



Machine F_4

Question 3: Machines F_3 and F_4 differ only slightly. What do they do, and how do they differ?

Give the result of taking the sentence **colorless # green # ideas # sleep # furiously** and:

- ...putting it through F_3.
- ...putting it through F_4.
- ...putting it through F_3, and putting the result through F_4.
- ...putting it through F_4, and putting the result through F_3.

Before actually putting the strings through, try to picture what the results will look like, and in particular how the first two will differ (if at all) and how the last two will differ (if at all). You don't have to write anything up; just give it some thought, and see if the results are what you expect.

(Note: in case it's not clear, "putting String X through Machine M" means "giving the output string Machine M associates with String X as the input string".)

2. PROPOSITIONAL LOGIC

2.1. Tautologies, Contradictions, etc.

Classify each of the following strings as either

- not a member of *PL*, the set of strings of propositional logic;
- a tautology, i.e. a formula that's necessarily true;
- a contradiction, i.e. a formula that's necessarily false; or
- simply a formula, but one whose truth or falsity depends on the truth and falsity of the propositions it's made of.

(compare *dead*, *good*, *evil*, and *unknown*, respectively)

Show your work.

a. **q**

b. **[p → [q → p]]**

c. **p ~ ∨ ~ q**

d. **[p ∨ ~p]**

e. **[[p ∧ q] → [p ∨ q]]**

f. **[p ∨ [q ∧ p]]**

2.2. Proofs

Classify each of the following proofs as either

- valid (whenever the premises are true, the conclusion is true), or
- invalid (the conclusion can be false even if the premises are true).

Show your work.

a. **[p → q]**
[q → r]
∴[p → r]

b. **[p ∨ q]**
[q ∨ r]
∴[p ∨ r]

c. **[~[p ∨ q]]**
∴[[~p] ∧ [~q]]

d. **[~[p ∧ q]]**
∴[[~p] ∨ [~q]]

e. **[p → q]**
q
∴p

f. **[[p → q] → [p → r]]**
∴[q → r]

3. PREDICATE LOGIC

3.1. Translating To and From Predicate Logic

Given the following meanings:

- **w**: Mrs. White
- **s**: Miss Scarlet
- **k**: Mrs. Peacock
- **p**: Professor Plum
- **m**: Colonel Mustard
- **g**: Mr. Green

- **B**: is in the ballroom
- **C**: is in the conservatory
- **H**: is in the hall
- **L**: is in the library
- **M**: is male
- **F**: is female

Translate (a)-(d) into predicate logic, and (e)-(h) into English.

a. Mrs. White is in the ballroom.
b. Mrs. Peacock is not in the hall.
c. Either Colonel Mustard is in the hall, or Colonel Mustard is in the library.
d. If Mr. Green is in the conservatory, then Miss Scarlet is in the conservatory.
e. **[~[Lg ∨ Hg]]**
f. **[Bw ∧ [Bp ∨ Bm]]**
g. **[[~Bk] → Ck]**
h. **[Hp ∧ [~Hp]]**

3.2. Evaluating Formulas of Predicate Logic

Now, given the following locations and the people in them:

Ballroom	*Hall*	*Library*
Mrs. White	Mrs. Peacock	Colonel Mustard
Miss Scarlet	Professor Plum	Mr. Green

and the information that White, Scarlet, and Peacock are female, and Plum, Mustard, and Green are male, evaluate the truth or falsity of the following. Explain your reasoning!

a. **[∀*x* [F*x* → B*x*]]**
b. **[∀*x* [B*x* → F*x*]]**
c. **[[∃*x* F*x*] ∧ [∃*x* L*x*]]**
d. **[∃*x* [F*x* ∧ L*x*]]**
e. **[∀*x* [M*x* → [H*x* ∨ L*x*]]]**
f. **[∀*x* [[M*x* → H*x*] ∨ [M*x* → L*x*]]]**