LING 106: Homework 4

Assigned: February 4, 2009
Due: February 20, 2009

1. DEVISING A FINITE STATE AUTOMATON

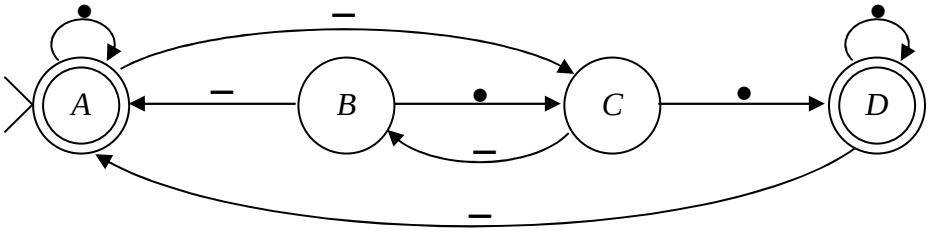
At home, I have a fan with two speeds, **LOW** and **HIGH**. It can be set to blow in either direction, **IN** or **OUT**. (The latter is useful for ventilation in a kitchen, for instance.)

The fan is electronic, so it doesn't have the usual dial that you set into "off", "low", or "high". Instead, it has three buttons: ***up***, ***down***, and ***reverse***. Pressing ***up*** takes the fan from **OFF** to **LOW** or from **LOW** to **HIGH** (nothing happens if the fan is already set to **HIGH**); pressing ***down*** does the opposite. If the fan is running, pressing ***reverse*** switches the fan from **IN** to **OUT** or vice versa. To my annoyance, when you turn the fan off, it doesn't remember which way it was blowing: when you turn it on, it always blows inward, even if it was blowing outward when you turned it off. (Pressing the ***reverse*** button when the fan is off doesn't do anything.)

a. Draw a deterministic finite state automaton that models the fan's behavior. (Assume that the fan starts in the off position, and isn't "done" until it's turned off again.)

b. Give two different strings of length six that the FSA accepts.

2. FORMAL DESCRIPTION OF A FSA



Finite State Automaton $\boldsymbol{M}_1$

Give the formal description of $\boldsymbol{M_1} = \langle Q, \Sigma, \delta, s, F \rangle$. As a starting point, $\Sigma = \{\bullet, -\}$.

Which of the following strings does $\boldsymbol{M_1}$ accept?

a. ● ● − −
b. ε
c. ● − ● − ●
d. ● ● ● − − − ● ● ●
e. ● −
f. − ● ●
g. − ● − − − ● − ●
h. ● ● ●
i. − − − −
j. ● − ● ● ●

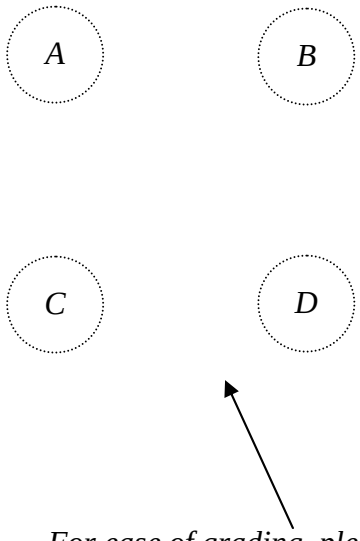
3. DIAGRAM OF AN FSA

Draw a diagram of $\boldsymbol{M_2}$ = <*Q*, Σ, δ, *s*, *F*>, where:

1. *Q* = {*A*, *B*, *C*, *D*}
2. Σ = {**s**, **l**, **a**}
3. δ is defined as

	s	**l**	**a**
A	*B*	*A*	*C*
B	*C*	*A*	*D*
C	*B*	*D*	*C*
D	*D*	*B*	*C*

4. *s* = *C*
5. *F* = {*C*}



For ease of grading, please arrange the states in this configuration.

Which of the following strings does $\boldsymbol{S_2}$ accept?

a. ε
b. **l**
c. **llsss**
d. **salsa**
e. **al**
f. **aaaa**
g. **asllaaaa**
h. **ssssss**
i. **lslls**
j. **sassla**

4. REGULAR LANGUAGES

Show that the following languages are regular. That is, construct diagrams for deterministic FSAs that model the following languages. (A diagram is fine; you don't need to give the 5-tuple.) For each language, Σ = {**0**, **1**}. Use as few states as possible.

a. {ε, **01**, **10**, **100**}
b. {*s* | the length of *s* is between 2 and 5, inclusive}
c. {*s* | *s* starts with **1**, or ends with **1**, or both}
d. {*s* | *s* contains the substring **00**}
e. {*s* | *s* starts with **0** and does not contain the substring **11**}
f. {*s* | every instance of **10** in *s* is immediately followed by an instance **01**}[1]

[1] e.g., **1**, **011**, **10011**, **001001001** are strings of the language; **10**, **10010**, **10001** are not. If you're still confused, feel free to email me a few strings, which I can classify as in the language or not.