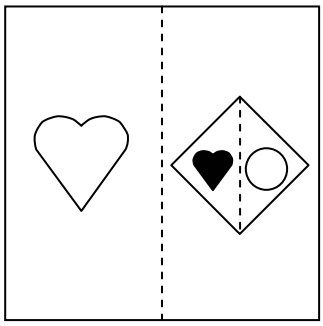
# LING 106: Homework 1

Assigned: Wednesday, Jan. 14, 2009
Due: Wednesday, Jan. 21, 2009

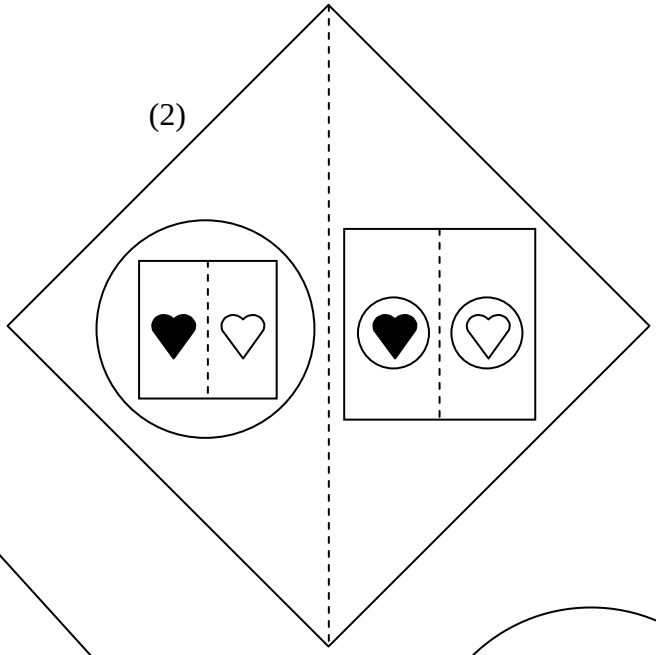
## 1. CATEGORIZING SHAPES

Categorize each of the following shapes as (a) dead, (b) good, (c) evil, or (d) unknown. Explain how you can tell. (Reasoning "outside the system" is fine for this problem.)

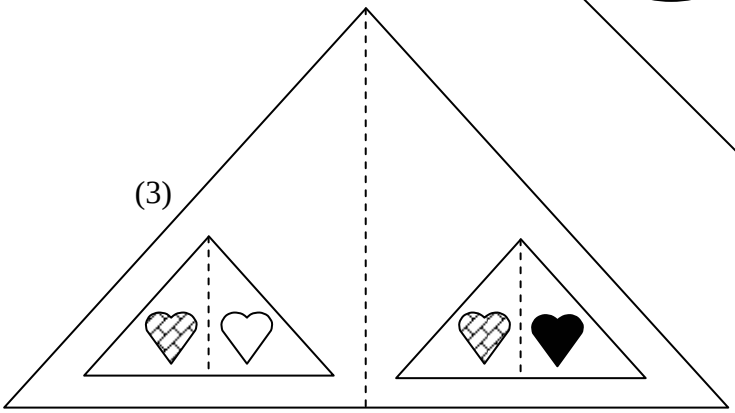
(1)



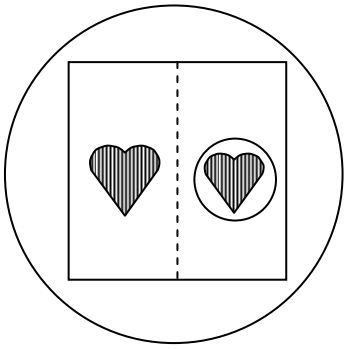
(2)



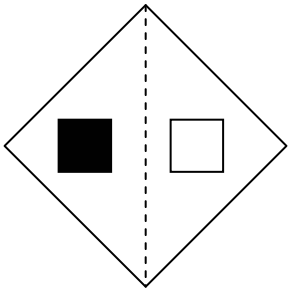
(3)



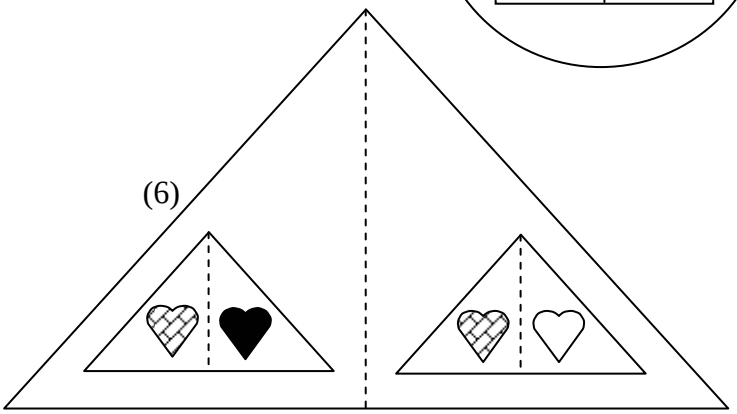
(4)



(5)



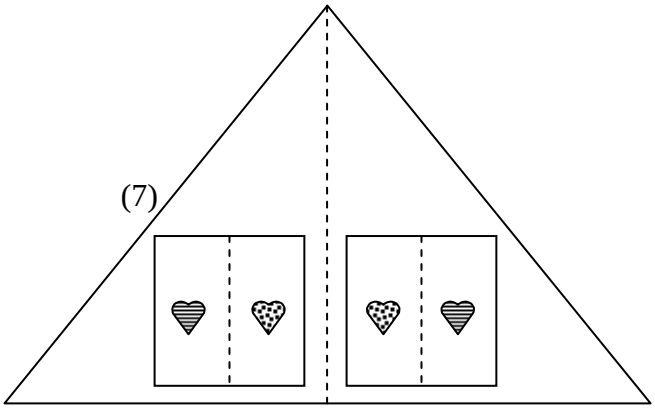
(6)



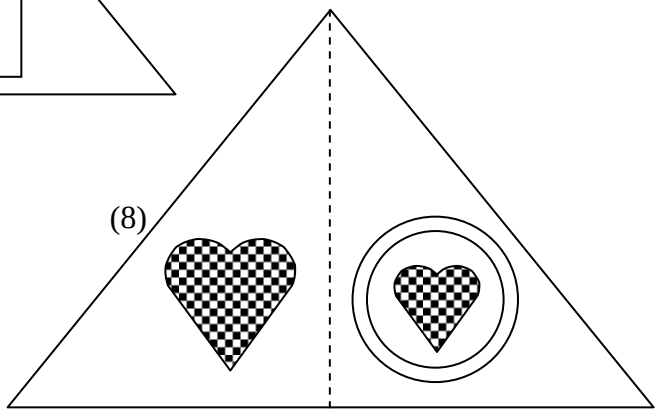
## 2. PROOFS

Each of the three triangles below is a good shape. Prove it, using the techniques in class—i.e., reasoning within the system (no "meta-reasoning" allowed here). Feel free to substitute colors or other patterns, as long as you do so consistently.

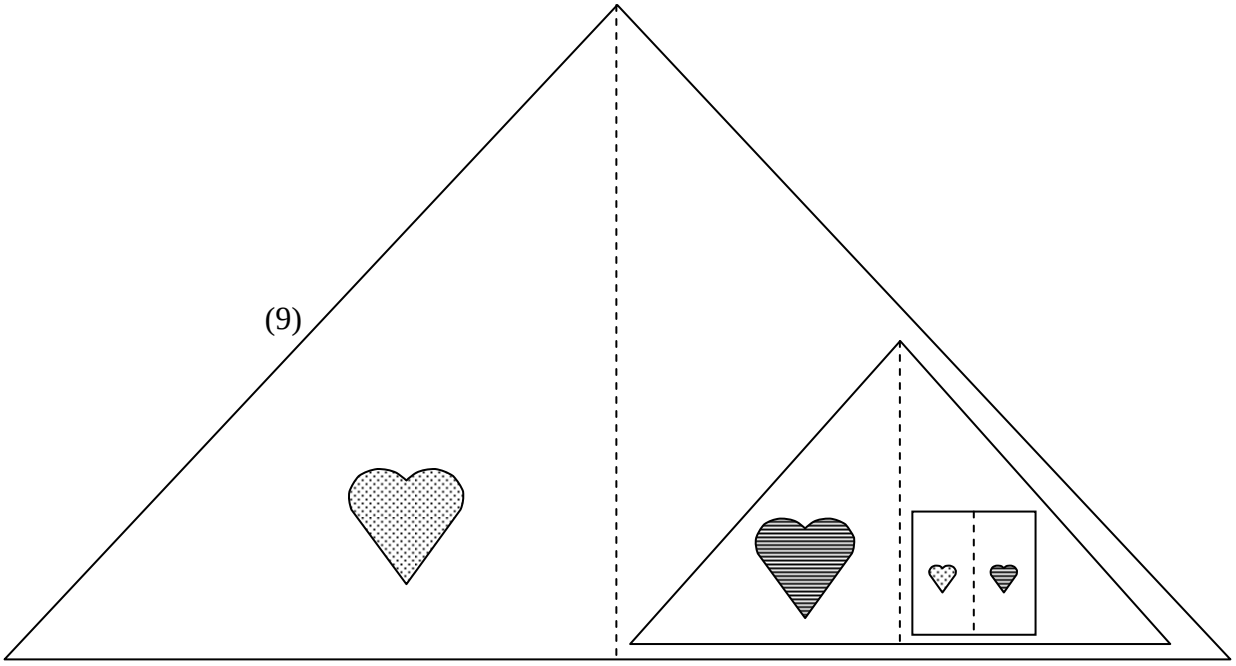
(7)



(8)



(9)



## 3. SPECIFYING SETS

In class, we saw three notations for specifying a set: list, predicate, and recursive. For each of the following sets: (a) express the set in the two other notations (e.g., for Set A, give the predicate notation and the recursive notation); (b) give the cardinality of the set.

- $A = \{\mathbf{a}, \mathbf{aaa}, \mathbf{aaaaa}, \mathbf{aaaaaaa}, \mathbf{aaaaaaaaa}, ...\}$
- $B = \{3, 7, 13, 17, 23, 27, 33, 37, 43, 47, ...\}$
- $C = \{x \mid x \text{ is an integer consisting of a 6 followed by some number of 0s}\}$
- $D$, such that
  - Maine $\in D$
  - If $y \in D$ and $x$ is a US state that borders $y$, then $x \in D$
  - Nothing else is a member of $D$

## 4. SET RELATIONS

Using the following sets, classify statements (a)-(u) as true or false.

$E = \{\mathbf{p}, \{\mathbf{q}\}, \mathbf{q}\}$ $F = \{\mathbf{p}, \mathbf{r}, \mathbf{s}, \{\mathbf{p}, \mathbf{q}\}\}$ $G = \{\mathbf{s}, \mathbf{p}, \{\varnothing\}\}$ $H = \{\mathbf{p}, \varnothing\}$

a. $\mathbf{p} \in E$
b. $\mathbf{q} \in E$
c. $\mathbf{q} \subseteq E$
d. $\{\mathbf{p}\} \in E$
e. $\{\mathbf{p}\} \subseteq E$
f. $\{\mathbf{q}\} \in E$
g. $\{\mathbf{q}\} \subseteq E$
h. $\{\mathbf{p},\mathbf{q}\} \in F$
i. $\{\mathbf{p},\mathbf{q}\} \subseteq F$
j. $\{\mathbf{p},\mathbf{s}\} \subseteq G$
k. $\{\mathbf{p},\mathbf{s}\} \subseteq H$
l. $\varnothing \in E$
m. $\varnothing \subseteq E$
n. $\varnothing \in G$
o. $\{\varnothing\} \in G$
p. $\varnothing \subseteq G$
q. $\{\varnothing\} \subseteq G$
r. $\varnothing \in H$
s. $\{\varnothing\} \in H$
t. $\varnothing \subseteq H$
u. $\{\varnothing\} \subseteq H$

## 5. SET OPERATIONS

Using the following sets, and taking the universe to be $I \cup J \cup K \cup L$, specify—in list notation—the sets in (a)-(j).

$I = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ $J = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}\}$ $K = \{\mathbf{a}, \{\mathbf{a}, \mathbf{b}\}\}$ $L = \{\mathbf{b}, \{\mathbf{g}\}, \{\mathbf{a}, \mathbf{b}\}\}$

a. $U$
b. $I \cup J$
c. $I \cap J$
d. $I - J$
e. $J - I$
f. $J \cap K$
g. $K - J$
h. $K \cup J'$
i. $L \cup J$
j. $L' \cup K'$
k. $\wp(K)$

## 6. RELATIONS AND FUNCTIONS

Given the following sets...

$W = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ $X = \{\mathbf{1}, \mathbf{2}, \mathbf{3}\}$ $Y = \{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ $Z = \{\mathbf{5}, \mathbf{6}, \mathbf{7}, \mathbf{8}\}$

...and the following relations, where $R$, $S$, $T$ are relations from $W$ to $X$ and $M$, $N$, $O$ are relations from $Y$ to $Z$...

$R = \{\langle\mathbf{a},\mathbf{1}\rangle, \langle\mathbf{c},\mathbf{3}\rangle, \langle\mathbf{d},\mathbf{2}\rangle\}$
$S = \{\langle\mathbf{a},\mathbf{2}\rangle, \langle\mathbf{b},\mathbf{1}\rangle, \langle\mathbf{c},\mathbf{1}\rangle, \langle\mathbf{d},\mathbf{3}\rangle\}$
$T = \{\langle\mathbf{a},\mathbf{1}\rangle, \langle\mathbf{b},\mathbf{2}\rangle, \langle\mathbf{d},\mathbf{3}\rangle, \langle\mathbf{a},\mathbf{2}\rangle\}$

$M = \{\langle\mathbf{e},\mathbf{5}\rangle, \langle\mathbf{f},\mathbf{7}\rangle, \langle\mathbf{g},\mathbf{6}\rangle\}$
$N = \{\langle\mathbf{e},\mathbf{6}\rangle, \langle\mathbf{f},\mathbf{5}\rangle, \langle\mathbf{g},\mathbf{5}\rangle\}$
$O = \{\langle\mathbf{e},\mathbf{5}\rangle, \langle\mathbf{f},\mathbf{6}\rangle, \langle\mathbf{g},\mathbf{7}\rangle, \langle\mathbf{e},\mathbf{8}\rangle\}$

then:

a. Compute $W \times X$ and $Y \times Z$.
b. State, for each of the six relations: (a) whether it is a function, and (b) if it is, whether it is onto or into, and whether it is one-to-one or many-to-one.