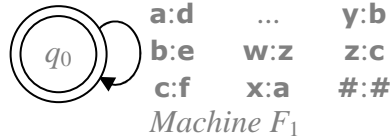


# LING 106: Homework 6

## 1. READING FINITE STATE TRANSDUERS

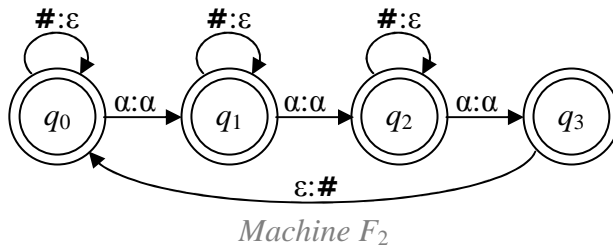
For example,  $F_1$ :



**Question 1:** What does  $F_1$  do? What is the output that the machine associates with the input **meet # me # at # the # library #**?

$F_1$  is a Caesar shifter,  $n = 3$ . (Spacing is preserved.)

**meet # me # at # the # library #  $\rightarrow$  phhw # ph # dw # wkh # oleudub #**



**Question 2:** What does  $F_2$  do?

$F_2$  regroups the letters into strings of length 3, separated by word breaks.

**meet # me # at # the # library #  $\rightarrow$  mee # tme # att # hel # ibr # ary #**

The entire input/output pair, with states and correspondences shown:

<b>m</b>	<b>e</b>	<b>e</b>	$\epsilon$	<b>t</b>	<b>#</b>	<b>m</b>	<b>e</b>	$\epsilon$	<b>#</b>	<b>a</b>	<b>t</b>	$\epsilon$	<b>h</b>	<b>e</b>	<b>#</b>	<b>l</b>	$\epsilon$	<b>i</b>	<b>b</b>	<b>r</b>	$\epsilon$	<b>a</b>	<b>r</b>	<b>y</b>	$\epsilon$	<b>#</b>																								
0		1		2		3		0		1		1		2		3		0		0		1		2		2		3		0		1		2		2		3		0		1		2		3		0		0
<b>m</b>	<b>e</b>	<b>e</b>	<b>#</b>	<b>t</b>	$\epsilon$	<b>m</b>	<b>e</b>	<b>#</b>	$\epsilon$	<b>a</b>	<b>t</b>	$\epsilon$	<b>t</b>	<b>#</b>	<b>h</b>	<b>e</b>	$\epsilon$	<b>l</b>	<b>#</b>	<b>i</b>	<b>b</b>	<b>r</b>	<b>#</b>	<b>a</b>	<b>r</b>	<b>y</b>	<b>#</b>	$\epsilon$																						



## 2. PROPOSITIONAL LOGIC

### 2.1. Tautologies, Contradictions, etc.

- |    |                                     |    |   |
|----|-------------------------------------|----|---|
| a. | $q$                                 | d. | $[p \vee \sim p]$                       |
| b. | $[p \rightarrow [q \rightarrow p]]$ | e. | $[[p \wedge q] \rightarrow [p \vee q]]$ |
| c. | $p \sim \vee \sim q$                | f. | $[p \vee [q \wedge p]]$                 |

a: formula	d: not well-formed ( $[p \vee [\sim p]]$ would be a tautology)
b: tautology	e: tautology
c: not well-formed	f: formula (actually equivalent in truth to $p$ )

$p$	$q$	$[q \rightarrow p]$	$[p \rightarrow [q \rightarrow p]]$	$p$	$q$	$[p \wedge q]$	$[p \vee q]$	$[[p \wedge q] \rightarrow [p \vee q]]$
T	T	T	T	T	T	T	T	T
T	F	T	T	T	F	F	T	T
F	T	F	T	F	T	F	T	T
F	F	T	T	F	F	F	F	T

$p$	$q$	$[q \wedge p]$	$[p \vee [q \wedge p]]$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

### 2.2. Proofs

a.	$[p \rightarrow q]$	$p$	$q$	$r$	$[p \rightarrow q]$	$[q \rightarrow r]$	$[p \rightarrow r]$	VALID
	$[q \rightarrow r]$	T	T	T	T	T	T	
	$\therefore [p \rightarrow r]$	T	T	F	T	F	F	
		T	F	T	F	T	T	
		T	F	F	F	T	F	
		F	T	T	T	T	T	
		F	T	F	T	F	T	
		F	F	T	T	T	T	
		F	F	F	T	T	T	

b.  $[p \vee q]$     $p$     $q$     $r$     $[p \vee q]$     $[q \vee r]$     $[p \vee r]$    INVALID

$[q \vee r]$	T	T	T	T	T	T
$\therefore [p \vee r]$	T	T	F	T	T	T
	T	F	T	T	T	T
	T	F	F	T	F	T
	F	T	T	T	T	T
	F	T	F	<b>T</b>	<b>T</b>	<b>F</b>
	F	F	T	F	T	T
	F	F	F	F	F	F

c.  $[\sim[p \vee q]]$     $p$     $q$     $[\sim p]$     $[\sim q]$     $[p \vee q]$     $[\sim[p \vee q]]$     $[[\sim p] \wedge [\sim q]]$    VALID

$\therefore [[\sim p] \wedge [\sim q]]$

	T	T	F	F	T	F	F
	T	F	F	T	T	F	F
	F	T	T	F	T	F	F
	F	F	T	T	F	<b>T</b>	<b>T</b>

d.  $[\sim[p \wedge q]]$     $p$     $q$     $[\sim p]$     $[\sim q]$     $[p \wedge q]$     $[\sim[p \wedge q]]$     $[[\sim p] \vee [\sim q]]$    VALID

$\therefore [[\sim p] \vee [\sim q]]$

	T	T	F	F	T	F	F
	T	F	F	T	F	<b>T</b>	<b>T</b>
	F	T	T	F	F	<b>T</b>	<b>T</b>
	F	F	T	T	F	<b>T</b>	<b>T</b>

e.  $[p \rightarrow q]$     $p$     $q$     $[p \rightarrow q]$     $p$    INVALID

$q$	T	T	T	T
$\therefore p$	T	F	F	T
	F	T	<b>T</b>	<b>F</b>
	F	F	T	F

f.  $[[p \rightarrow q] \rightarrow [p \rightarrow r]]$   
 $\therefore [q \rightarrow r]$

$p$	$q$	$r$	$[p \rightarrow q]$	$[p \rightarrow r]$	$[[p \rightarrow q] \rightarrow [p \rightarrow r]]$	$[q \rightarrow r]$	INVALID
T	T	T	T	T	T	T	
T	T	F	T	F	F	F	
T	F	T	F	T	T	T	
T	F	F	F	F	T	T	
F	T	T	T	T	T	T	
F	T	F	T	T	<b>T</b>	<b>F</b>	
F	F	T	T	T	T	T	
F	F	F	T	T	T	T	

### 3. PREDICATE LOGIC

#### 3.1. Translating To and From Predicate Logic

- a. Mrs. White is in the ballroom.
- b. Mrs. Peacock is not in the hall.
- c. Either Colonel Mustard is in the hall, or Colonel Mustard is in the library.
- d. If Mr. Green is in the conservatory, then Miss Scarlet is in the conservatory.
- e.  $[\sim[\mathbf{Lg} \vee \mathbf{Hg}]]$
- f.  $[\mathbf{Bw} \wedge [\mathbf{Bp} \vee \mathbf{Bm}]]$
- g.  $[[\sim\mathbf{Bk}] \rightarrow \mathbf{Ck}]$
- h.  $[\mathbf{Hp} \wedge [\sim\mathbf{Hp}]]$

- a:  $\mathbf{Bw}$                       e: Mr. Green is in neither the library nor the hall.
- b:  $[\sim\mathbf{Hk}]$                     f: Mrs. White is in the ballroom, and either Prof. Plum or Col. Mustard is too.
- c:  $[\mathbf{Hm} \vee \mathbf{Lm}]$             g: If Mrs. Peacock is not in the ballroom, then she's in the conservatory.
- d:  $[\mathbf{Cg} \rightarrow \mathbf{Cs}]$             h: Professor Plum is in the hall, and Professor Plum is not in the hall.

#### 3.2. Evaluating Formulas of Predicate Logic

Ballroom  
Mrs. White  
Miss Scarlet

Hall  
Mrs. Peacock  
Professor Plum

Library  
Colonel Mustard  
Mr. Green

- a.  $[\forall x [\mathbf{Fx} \rightarrow \mathbf{Bx}]]$
- b.  $[\forall x [\mathbf{Bx} \rightarrow \mathbf{Fx}]]$
- c.  $[[\exists x \mathbf{Fx}] \wedge [\exists x \mathbf{Lx}]]$
- d.  $[\exists x [\mathbf{Fx} \wedge \mathbf{Lx}]]$
- e.  $[\forall x [\mathbf{Mx} \rightarrow [\mathbf{Hx} \vee \mathbf{Lx}]]]$
- f.  $[\forall x [[\mathbf{Mx} \rightarrow \mathbf{Hx}] \vee [\mathbf{Mx} \rightarrow \mathbf{Lx}]]]$

- a: False. There's an individual for whom  $[\mathbf{Fx} \rightarrow \mathbf{Bx}]$  is false, namely Mrs. Peacock.
- b: True. If  $x$  is in the ballroom, then  $x$  is female, for all  $x$ .
- c: True. Both conjuncts are true: there is someone female (e.g., Mrs. White) and there is someone in the library (e.g., Mr. Green).
- d: False. There's no single  $x$  who makes the conjunction true.
- e: True. For all individuals, if they're male, it's true that they're in either the hall or the library.
- f: True. for all individuals, it's either true that if they're male, they're in the hall, or that if they're male, they're in the library.