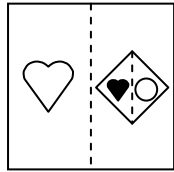


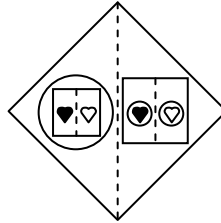
LING 106: Homework 1 Answers

1. CATEGORIZING SHAPES



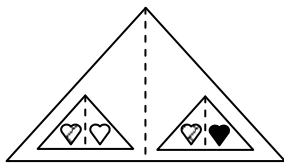
1. Dead

A circle with nothing in it is dead, so the diamond doesn't have two living shapes in it (and is thus a dead shape), so the square doesn't have two living shapes in it (and is thus a dead shape).



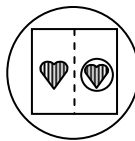
2. Good

On the left side of the diamond is a circle with a square in it. It's not the case that both halves of the square contain good shapes—the empty heart is evil. So that square is evil (SR); therefore, that square inside a circle is good (CR). So at least one of the two shapes in the diamond is a good shape (the other happens to be evil); therefore the diamond is a good shape (DR).



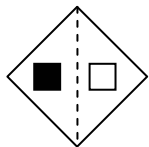
3. Good

Take the triangle on the right side. The shape on its right side is good (HR), so that triangle is good (TR). Therefore the right-hand shape in the overall triangle is good, so the whole triangle is good (TR).



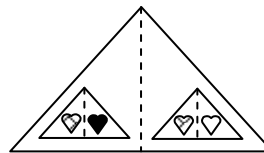
4. Good

Consider the square. If the striped heart is evil, then the square's left shape is evil; if the striped heart is good, then the circle with the heart in it is evil, so the square's right shape is evil. Therefore, it can't be the case that both shapes in the square are good; so the square must be evil (SR). Therefore, the whole shape is an evil shape inside a circle, which makes it a good shape (CR).



5. Dead

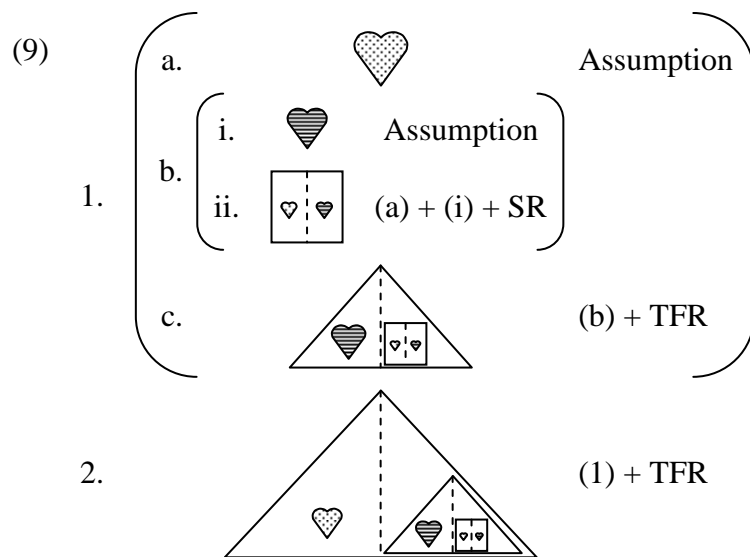
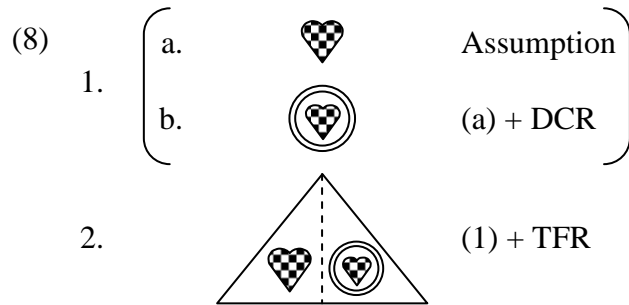
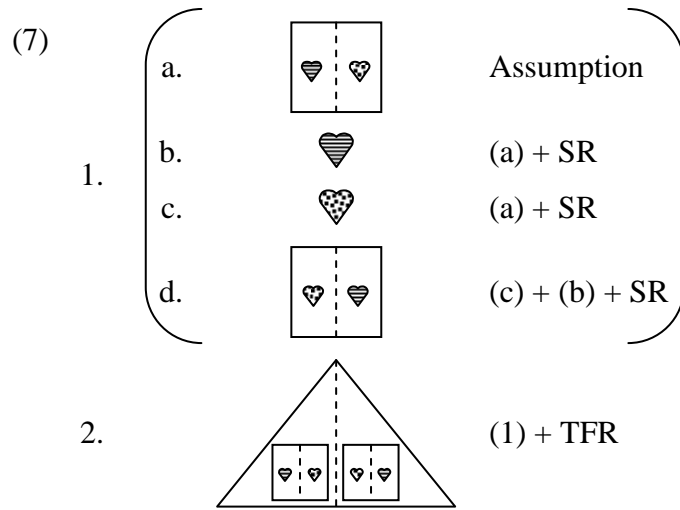
Squares can't be shaded or unshaded; they're containers for hearts that are shaded or unshaded.



6. Unknown

The triangle on the left is good (see #3). So if the triangle on the right is good, the whole shape is good; and if it's evil, the whole shape is evil. Looking at the triangle on the right: its right-hand shape is evil; so if the patterned heart is good, that triangle is evil; and if it's evil, that triangle is good. But since the patterned shape is unknown, we can't know whether the right-hand triangle is good or evil. So the overall shape is also unknown.

2. PROOFS



3. SPECIFYING SETS

- $A = \{\mathbf{a}, \mathbf{aaa}, \mathbf{aaaaa}, \mathbf{aaaaaaa}, \mathbf{aaaaaaaaa}, \dots\}$

- $A = \{x \mid x \text{ consists of an odd number of } \mathbf{a}\mathbf{s}\}$
- $\mathbf{a} \in A$; if $x \in A$, then $x\mathbf{aa} \in A$; nothing else is in A
- $|A| = \infty$

- $B = \{3, 7, 13, 17, 23, 27, 33, 37, 43, 47, \dots\}$

- $B = \{x \mid x = 10n+3 \text{ or } 10n+7 \text{ for } 0 \leq n\}$
- $3 \in B, 7 \in B$; if $x \in B$, then $x+10 \in B$; nothing else is in B
- $|B| = \infty$

- $C = \{x \mid x \text{ is an integer consisting of a 6 followed by some number of 0s}\}$

- $C = \{6, 60, 600, 6000, 60000, \dots\}$
- $6 \in C$; if $x \in C$, then $10 \cdot x \in C$; nothing else is in C
- $|C| = \infty$

- D , such that $\text{Maine} \in D \dots$

- $D = \{x \mid x \text{ is one of the Lower 48 US states}\}$ (or “a state in the continental US”, etc.)
- $D = \{$

Alabama,	Arizona,	Arkansas,	California,	Colorado,
Connecticut,	Delaware,	Florida,	Georgia,	Idaho,
Illinois,	Indiana,	Iowa,	Kansas,	Kentucky,
Louisiana,	Maine,	Maryland,	Massachusetts,	Michigan,
Minnesota,	Mississippi,	Missouri,	Montana,	Nebraska,
Nevada,	New Hampshire,	New Jersey,	New Mexico,	New York,
North Carolina,	North Dakota,	Ohio,	Oklahoma,	Oregon,
Pennsylvania,	Rhode Island,	South Carolina,	South Dakota,	Tennessee,
Texas,	Utah,	Vermont,	Virginia,	Washington,
West Virginia,	Wisconsin,	Wyoming}		
- *or:* $D = \{\text{Alabama, Arizona, Arkansas, ..., Georgia, Idaho, ..., Wisconsin, Wyoming}\}$
- $|D| = 48$

4. SET RELATIONS

$$E = \{\mathbf{p}, \{\mathbf{q}\}, \mathbf{q}\} \quad F = \{\mathbf{p}, \mathbf{r}, \mathbf{s}, \{\mathbf{p}, \mathbf{q}\}\} \quad G = \{\mathbf{s}, \mathbf{p}, \{\emptyset\}\} \quad H = \{\mathbf{p}, \emptyset\}$$

- | | | | | | | | | |
|----|------------------------------|----------|----|--|----------|----|-----------------------------|----------|
| a. | $\mathbf{p} \in E$ | T | h. | $\{\mathbf{p}, \mathbf{q}\} \in F$ | T | o. | $\{\emptyset\} \in G$ | T |
| b. | $\mathbf{q} \in E$ | T | i. | $\{\mathbf{p}, \mathbf{q}\} \subseteq F$ | F | p. | $\emptyset \subseteq G$ | T |
| c. | $\mathbf{q} \subseteq E$ | F | j. | $\{\mathbf{p}, \mathbf{s}\} \subseteq G$ | T | q. | $\{\emptyset\} \subseteq G$ | F |
| d. | $\{\mathbf{p}\} \in E$ | F | k. | $\{\mathbf{p}, \mathbf{s}\} \subseteq H$ | F | r. | $\emptyset \in H$ | T |
| e. | $\{\mathbf{p}\} \subseteq E$ | T | l. | $\emptyset \in E$ | F | s. | $\{\emptyset\} \in H$ | F |
| f. | $\{\mathbf{q}\} \in E$ | T | m. | $\emptyset \subseteq E$ | T | t. | $\emptyset \subseteq H$ | T |
| g. | $\{\mathbf{q}\} \subseteq E$ | T | n. | $\emptyset \in G$ | F | u. | $\{\emptyset\} \subseteq H$ | T |

5. SET OPERATIONS

$$I = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\} \quad J = \{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}\} \quad K = \{\mathbf{a}, \{\mathbf{a}, \mathbf{b}\}\} \quad L = \{\mathbf{b}, \{\mathbf{g}\}, \{\mathbf{a}, \mathbf{b}\}\}$$

- | | | |
|----|--------------|--|
| a. | U | $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}, \{\mathbf{g}\}, \{\mathbf{a}, \mathbf{b}\}\}$ |
| b. | $I \cup J$ | $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}\}$ |
| c. | $I \cap J$ | $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}\}$ |
| d. | $I - J$ | \emptyset |
| e. | $J - I$ | $\{\mathbf{e}, \mathbf{f}, \mathbf{g}\}$ |
| f. | $J \cap K$ | $\{\mathbf{a}\}$ |
| g. | $K - J$ | $\{\{\mathbf{a}, \mathbf{b}\}\}$ |
| h. | $K \cup J'$ | $\{\mathbf{a}, \{\mathbf{g}\}, \{\mathbf{a}, \mathbf{b}\}\}$ |
| i. | $L \cup J$ | $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}, \{\mathbf{g}\}, \{\mathbf{a}, \mathbf{b}\}\}$ |
| j. | $L' \cup K'$ | $\{\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}, \mathbf{g}, \{\mathbf{g}\}\}$ |
| k. | $\wp(K)$ | $\{\{\}, \{\mathbf{a}\}, \{\{\mathbf{a}, \mathbf{b}\}\}, \{\mathbf{a}, \{\mathbf{a}, \mathbf{b}\}\}\}$ |

6. RELATIONS AND FUNCTIONS

$$W = \{a, b, c, d\} \quad X = \{1, 2, 3\} \quad Y = \{e, f, g\} \quad Z = \{5, 6, 7, 8\}$$

$$R = \{\langle a, 1 \rangle, \langle c, 3 \rangle, \langle d, 2 \rangle\}$$

$$S = \{\langle a, 2 \rangle, \langle b, 1 \rangle, \langle c, 1 \rangle, \langle d, 3 \rangle\}$$

$$T = \{\langle a, 1 \rangle, \langle b, 2 \rangle, \langle d, 3 \rangle, \langle a, 2 \rangle\}$$

$$M = \{\langle e, 5 \rangle, \langle f, 7 \rangle, \langle g, 6 \rangle\}$$

$$N = \{\langle e, 6 \rangle, \langle f, 5 \rangle, \langle g, 5 \rangle\}$$

$$O = \{\langle e, 5 \rangle, \langle f, 6 \rangle, \langle g, 7 \rangle, \langle e, 8 \rangle\}$$

a. $W \times X = \{ \langle a, 1 \rangle, \langle a, 2 \rangle, \langle a, 3 \rangle, \langle b, 1 \rangle, \langle b, 2 \rangle, \langle b, 3 \rangle, \langle c, 1 \rangle, \langle c, 2 \rangle, \langle c, 3 \rangle, \langle d, 1 \rangle, \langle d, 2 \rangle, \langle d, 3 \rangle \}$

$$Y \times Z = \{ \langle e, 5 \rangle, \langle e, 6 \rangle, \langle e, 7 \rangle, \langle e, 8 \rangle, \langle f, 5 \rangle, \langle f, 6 \rangle, \langle f, 7 \rangle, \langle f, 8 \rangle, \langle g, 5 \rangle, \langle g, 6 \rangle, \langle g, 7 \rangle, \langle g, 8 \rangle \}$$

- b.
- | | | | |
|-------|-----------------------------|-------|-----------------------------|
| R : | not a function | M : | function, into, one-to-one |
| S : | function, onto, many-to-one | N : | function, into, many-to-one |
| T : | not a function | O : | not a function |