Algebraic Semantics Redux

Script to the DoKo Summer School lecture

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Lecture 1

Mereology: basic concepts

1.1 Introduction

- Mereology: the study of parthood in philosophy and mathematical logic
- Mereology can be axiomatized in a way that gives rise to algebraic structures (sets with binary operations defined on them)

Figure 1.1: An algebraic structure

- Algebraic semantics: the branch of formal semantics that uses algebraic structures and parthood relations to model various phenomena
1.2 Mereology

- Basic motivation (Link 1998): entailment relation between collections and their members

(1)  a. John and Mary sleep. ⇒
    John sleeps and Mary sleeps.

   b. The water in my cup evaporated. ⇒
    The water at the bottom of my cup evaporated.

- Basic relation ≤ (parthood) – no consensus on what exactly it expresses

- Table 1.1 gives a few interpretations of the relation ≤ in algebraic semantics

**Table 1.1: Examples of unstructured parthood**

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>some horses</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a quantity of water</td>
<td>a portion of it</td>
</tr>
<tr>
<td>John, Mary and Bill</td>
<td>John</td>
</tr>
<tr>
<td>some jumping events</td>
<td>a subset of them</td>
</tr>
<tr>
<td>a running event from A to B</td>
<td>its part from A halfway towards B</td>
</tr>
<tr>
<td>a temporal interval</td>
<td>its initial half</td>
</tr>
<tr>
<td>a spatial interval</td>
<td>its northern half</td>
</tr>
</tbody>
</table>

**Table 1.2: Examples of structured parthood from Simons (1987)**

<table>
<thead>
<tr>
<th>Whole</th>
<th>Part</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (certain) man</td>
<td>his head</td>
</tr>
<tr>
<td>a (certain) tree</td>
<td>its trunk</td>
</tr>
<tr>
<td>a house</td>
<td>its roof</td>
</tr>
<tr>
<td>a mountain</td>
<td>its summit</td>
</tr>
<tr>
<td>a battle</td>
<td>its opening shot</td>
</tr>
<tr>
<td>an insect’s life</td>
<td>its larval stage</td>
</tr>
<tr>
<td>a novel</td>
<td>its first chapter</td>
</tr>
</tbody>
</table>

- All these are instances of unstructured parthood (arbitrary slices).
• Compare this with structured parthood (Simons 1987; Fine 1999; Varzi 2010) in Table 1.2 (cognitively salient parts).

• In algebraic semantics one usually models only unstructured parthood.

• Mereology started as an alternative to set theory; instead of $\in$ and $\subseteq$ there is only $\leq$.

• In algebraic semantics, mereology and set theory coexist.

• The most common axiom system is classical extensional mereology (CEM).

• The order-theoretic axiomatization of CEM starts with $\leq$ as a partial order (a reflexive, transitive, and symmetric relation).

• The proper-part relation $< \text{ restricts parthood to nonequal pairs.}$

• Sums are that which you get when you put several parts together.

• In CEM, every nonempty set $P$ has a unique sum $\biguplus P$.

• As a shorthand for binary sum, we write $\biguplus \{x, y\}$ as $x \oplus y$.

• Two applications of sum in linguistics are conjoined terms and definite descriptions.
  
  - For Sharvy (1980), $[\text{the water}] = \biguplus \text{water}$
  - For Link (1983), $[\text{John and Mary}] = j \oplus m$

• Another application: natural kinds as sums; e.g. the kind potato is $\biguplus \text{potato}$.

• But this needs to be refined for uninstantiated kinds such as dodo and phlogiston. One answer: kinds are individual concepts of sums (Chierchia 1998b). See Carlson (1977) and Pearson (2009) on kinds more generally.

1.3 Mereology and set theory

• Models of CEM (or “mereologies”) are essentially isomorphic to complete Boolean algebras with the bottom element removed, or equivalently complete semilattices with their bottom element removed (Tarski 1935; Pontow and Schubert 2006).

• Example: the powerset of a given set, with the empty set removed, and with the partial order given by the subset relation.

• CEM parthood is very similar to the subset relation (Table 1.3).
Table 1.3: Correlations between CEM and set theory

<table>
<thead>
<tr>
<th>Property</th>
<th>CEM</th>
<th>Set theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Reflexivity</td>
<td>$x \leq x$</td>
<td>$x \subseteq x$</td>
</tr>
<tr>
<td>2 Transitivity</td>
<td>$x \leq y \land y \leq z \rightarrow x \leq z$</td>
<td>$x \subseteq y \land y \subseteq z \rightarrow x \subseteq z$</td>
</tr>
<tr>
<td>3 Antisymmetry</td>
<td>$x \leq y \land y \leq x \rightarrow x = y$</td>
<td>$x \subseteq y \land y \subseteq x \rightarrow x = y$</td>
</tr>
<tr>
<td>4 Interdefinability</td>
<td>$x \leq y \leftrightarrow x \oplus y = y$</td>
<td>$x \subseteq y \leftrightarrow x \cup y = y$</td>
</tr>
<tr>
<td>5 Unique sum/union</td>
<td>$P \neq \emptyset \rightarrow \exists! z \text{ sum}(z, P)$</td>
<td>$\exists! z = \bigcup P$</td>
</tr>
<tr>
<td>6 Associativity</td>
<td>$x \oplus (y \oplus z) = (x \oplus y) \oplus z$</td>
<td>$x \cup (y \cup z) = (x \cup y) \cup z$</td>
</tr>
<tr>
<td>7 Commutativity</td>
<td>$x \oplus y = y \oplus x$</td>
<td>$x \cup y = y \cup x$</td>
</tr>
<tr>
<td>8 Idempotence</td>
<td>$x \oplus x = x$</td>
<td>$x \cup x = x$</td>
</tr>
<tr>
<td>9 Unique separation</td>
<td>$x &lt; y \rightarrow$</td>
<td>$x \subseteq y \rightarrow \exists! z[z = y - x]$</td>
</tr>
</tbody>
</table>

1.4 Selected literature

- An extended version of this script, including a complete axiomatic presentation of CEM, is at www.ling.upenn.edu/~champoll/tuebingen_mereology_script_1up.pdf.

- Textbooks:
  * Mathematical foundations: Partee, ter Meulen, and Wall (1990)
  * Algebraic semantics: Landman (1991)

- Books on algebraic semantics: Link (1998); Landman (2000)

- Seminal articles on algebraic semantics: Landman (1996); Krifka (1998)

- Mereology surveys: Simons (1987); Casati and Varzi (1999); Varzi (2010)
Lecture 2

Nouns: count, plural, mass

2.1 Algebraic closure and the plural

• Link (1983) has proposed algebraic closure as underlying the meaning of the plural.

(1)  
   a. John is a boy.  
   b. Bill is a boy.  
   c. ⇒ John and Bill are boys.

• Algebraic closure closes any predicate (or set) \( P \) under sum formation:

   (2)  \textbf{Definition: Algebraic closure (Link 1983)}

   The algebraic closure \( ^ * P \) of a set \( P \) is defined as \( \{ x \mid \exists P' \subseteq P \left[ x = \bigoplus P' \right] \} \).
   (This is the set that contains any sum of things taken from \( P \).)

• Link translates the argument in (i) as follows:

(3)  \( \text{boy}(j) \land \text{boy}(b) \Rightarrow ^*\text{boy}(j \oplus b) \)

• This argument is valid. Proof: From \( \text{boy}(j) \land \text{boy}(b) \) it follows that \( \{ j, b \} \subseteq \text{boy} \). Hence \( \exists P' \subseteq \text{boy}[j \oplus b = \bigoplus P'] \), from which we have \(^*\text{boy}(j \oplus b) \) by definition.

• There are different views on the meaning of the plural:

  – Exclusive view: the plural form \( N_{pl} \) essentially means the same as \textit{two or more} \( N \) (Link 1983; Chierchia 1998a).

(4)  \( [N_{pl}] = ^*[N_{sg}] - [N_{sg}] \)
(5) a. $[\text{boy}] = \{a, b, c\}$
b. $[\text{boys}] = *[\text{boy}] - [\text{boy}] = \{a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$

- Inclusive view: the plural form essentially means the same as one or more $N$ (Krifka 1986; Sauerland 2003; Sauerland, Anderssen, and Yatsushiro 2005; Chierchia 2010); the singular form blocks the plural form via competition

(6) $[N_{pl}] = *[N_{sg}]$

(7) a. $[\text{boy}] = \{a, b, c\}$
b. $[\text{boys}] = *[\text{boy}] = \{a, b, c, a \oplus b, b \oplus c, a \oplus c, a \oplus b \oplus c\}$

- Mixed view: plural forms are ambiguous between one or more $N$ and two or more $N$ (Farkas and de Swart 2010).

**Figure 2.1:** Different views on the plural.

• On both the inclusive and exclusive view, plural nouns are cumulative.

(8) **Definition:** Cumulative reference

$\text{CUM}(P) \overset{df}{=} \forall x[P(x) \rightarrow \forall y[P(y) \rightarrow P(x \oplus y)]]$

(A predicate $P$ is cumulative if and only if whenever it holds of two things, it also holds of their sum.)
• Problem for the exclusive view (Schwarzschild 1996, p. 5): downward entailing contexts

(9)  a. No doctors are in the room.
    b. Are there doctors in the room?

• Problem for the inclusive view: needs to be complemented by a blocking/implicature story

(10) *John is doctors.

• Problem for both views: dependent plurals (de Mey 1981)

(11) Five boys flew kites.
    ≈ Five boys flew two or more kites. ✓ exclusive
    (on the scopeless reading, requires two or more kites in total to be flown)
    ⇝ Inclusive view would make a wrong prediction.

(12) No boys flew kites.
    ≈ No boys flew one or more kites. ✓ inclusive
    ≠ No boys flew two or more kites. *exclusive
    (because one kite flown by a boy already falsifies the sentence)
    ⇝ Exclusive view would make a wrong prediction.

• Possible solution (Zweig 2008, 2009): take the inclusive view and treat the inference that at least two kites are flown as a grammaticalized scalar implicature. Scalar implicatures only surface when they strengthen the meaning of the sentence, as in (11), but not as in (12).

2.2 Singular count nouns

• Counting involves mapping to numbers. Let a “singular individual” be something which is mapped to the number 1, something to which we can refer by using a singular noun.

• One can assume that all singular individuals are atoms: the cat’s leg is not a part of the cat.

(13) Definition: Atom
    Atom(x) ≡ ¬∃y[y < x]
    (An atom is something which has no proper parts.)
Definition: Atomic part
\[ x \leq_{\text{Atom}} y \overset{\text{def}}{=} x \leq y \land \text{Atom}(x) \]
(Being an atomic part means being atomic and being a part.)

- Group nouns like committee, army, league have given rise to two theories.
  - Atomic theory: the entities in the denotation of singular group nouns are mereological atoms like other singular count nouns (Barker 1992; Schwarzschild 1996; Winter 2001)
  - Plurality theory: they are plural individuals (e.g. Bennett 1974)

- The question is whether the relation between a committee and its members is linguistically relevant, and if so whether it is mereological parthood.
- One can also assume that singular count nouns apply to “natural units” (Krifka 1989) that may nevertheless have parts, or that there are two kinds of parthood involved (Link 1983).
- If one allows for nonatomic singular individuals, one might still want to state that all singular count nouns have quantized reference (Krifka 1989): the cat’s leg is not itself a cat.

Definition: Quantized reference
\[ \text{QUA}(P) \overset{\text{def}}{=} \forall x[P(x) \rightarrow \forall y[y < x \rightarrow \neg P(y)]] \]
(A predicate \( P \) is quantized if and only if whenever it holds of something, it does not hold of any its proper parts.)

Exercise 2.1 Do you think that all nouns have quantized reference? Can you think of counterexamples?

2.3 Mass nouns and atomicity

- Mass nouns are compatible with the quantifiers much and little and reject quantifiers such as each, every, several, a/an, some and numerals (Bunt 2006; Chierchia 2010)
- Many nouns can be used as count nouns or as mass nouns:

  (16) a. Kim put an apple into the salad.
  b. Kim put apple into the salad.

- Mass nouns (if analyzed as predicates) have cumulative reference: add water to water and you get water
• In this, they parallel plural nouns (Link 1983)
• Mass nouns have been proposed to have divisive reference (Cheng 1973)

\[
\text{(17) Definition: Divisive reference} \\
\text{DIV}(P) \overset{\text{def}}{=} \forall x[P(x) \to \forall y[y < x \to P(y)]]
\]
(A predicate \( P \) is divisive if and only if whenever it holds of something, it also holds of each of its proper parts.)

Exercise 2.2 Do you think all mass nouns have divisive reference? Can you think of counterexamples?

• What comes first: the count-mass distinction or the individual-substance distinction? Quine (1960) claims the former. Acquisition evidence suggests the latter (Spelke 1990).
• We can also talk about atoms in general, by adding one of the following axioms to CEM:

\[
\text{(18) Optional axiom: Atomicity} \\
\forall y \exists x[x \leq_{\text{Atom}} y]
\]
(All things have atomic parts.)

\[
\text{(19) Optional axiom: Atomlessness} \\
\forall x \exists y[y < x]
\]
(All things have proper parts.)

• Of course, we don’t need to add either axiom. This is one of the advantages of mereology.
• Do count nouns always involve reference to atomic domains?
  – If yes: What about twig type nouns and group nouns?
  – If no: What determines if a concept is realized as a count noun?

• Do mass nouns ever involve reference to atomic domains? What about “fake mass nouns” (collective mass nouns like furniture, mail, offspring) (Barner and Snedeker 2005; Doetjes 1997; Chierchia 2010)?
  – If yes: why can’t you say “three furniture(s)?
  – If no: why do the parts of furniture not qualify as furniture?
Lecture 3

Verbs, events, and aspectual composition

3.1 Introduction

• Early work represents the meaning of a verb with \( n \) syntactic arguments as an \( n \)-ary relation

• Davidson (1967) argued that verbs denote relations between events and their arguments

• The neo-Davidsonian position (e.g. Carlson 1984; Parsons 1990; Schein 1993) relates the relationship between events and their arguments by thematic roles

• There are also intermediate positions, such as Kratzer (2000)

<table>
<thead>
<tr>
<th>Position</th>
<th>Verbal denotation</th>
<th>Example: Brutus stabbed Caesar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traditional</td>
<td>( \lambda y \lambda x [\text{stab}(x, y)] )</td>
<td>\text{stab}(b, c)</td>
</tr>
<tr>
<td>Classical Davidsonian</td>
<td>( \lambda y \lambda x \lambda e [\text{stab}(e, x, y)] )</td>
<td>( \exists e [\text{stab}(e, b, c)] )</td>
</tr>
<tr>
<td>Neo-Davidsonian</td>
<td>( \lambda e [\text{stab}(e)] )</td>
<td>( \exists e [\text{stab}(e) \land \text{ag}(e, b) \land \text{th}(e, c)] )</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>( \lambda y \lambda e [\text{stab}(e, y)] )</td>
<td>( \exists e [\text{ag}(e, b) \land \text{stab}(e, c)] )</td>
</tr>
</tbody>
</table>

• The Neo-Davidsonian position makes it easier to state generalizations across the categories of nouns and verbs, and to place constraints on thematic roles.

• Events are things like Jones’ buttering of the toast, Brutus’ stabbing of Caesar.

• Events form a mereology, so they include plural events (Bach 1986; Krifka 1998)

• Events can be both spatially and temporally extended, unlike intervals.
3.2 Thematic roles

- Thematic roles represent ways entities take part in events (Parsons 1990; Dowty 1991)
- Two common views:
  - Traditional view: thematic roles encapsulate generalizations over shared entailments of argument positions in different predicates (Gruber 1965; Jackendoft 1972)
    * agent (initiates the event, or is responsible for the event)
    * theme (undergoes the event)
    * instrument (used to perform an event)
    * sometimes also location and time
  - Alternative view: thematic roles as verb-specific relations: Brutus is not the agent of the stabbing event but the stabber (Marantz 1984). But this misses generalizations. And what about subjects of coordinated sentences like *A girl sang and danced*?

- No consensus on the inventory of thematic roles, but see Levin (1993) and Kipper-Schuler (2005) for wide-coverage role lists of English verbs

Questions:

- Do thematic roles have syntactic counterparts, the theta roles (something like silent prepositions)? Generative syntax says yes at least for agent: the “little v” head (Chomsky 1995)
- Does each verbal argument correspond to exactly one role (Chomsky 1981) or is the subject of a verb like *fall* both its agent and its theme (Parsons 1990)?
- Thematic uniqueness / Unique Role Requirement: Does each event have at most one agent, at most one theme etc. (widely accepted in semantics, see Carlson (1984, 1998); Parsons (1990); Landman (2000)) or no (Krifka (1992): one can touch both a man and his shoulder in the same event)?
- Are thematic roles their own algebraic closures (Krifka 1986, 1998; Landman 2000)?

(1) **Cumulativity assumption for thematic roles**

For any thematic role \( \theta \) it holds that \( \theta = \theta^\ast \). This entails that

\[
\forall e, e', x, y [\theta(e) = x \land \theta(e') = y \rightarrow \theta(e \oplus e') = x \oplus y]
\]

- I assume the answer is yes (makes things easier to formalize)
– To symbolize this, instead of writing th, I will write *th.

• As a consequence of (1), thematic roles are homomorphisms with respect to the $\oplus$ operation (that is, it doesn’t matter whether we first apply $\oplus$ or first the thematic role)

(2) Fact: Thematic roles are sum homomorphisms

For any thematic role $\theta$, it holds that $\theta(e \oplus e') = \theta(e) \oplus \theta(e')$.

(The $\theta$ of the sum of two events is the sum of their $\theta$s.)

• Potential challenge to this assumption: the rosebush story (Kratzer 2003). Suppose there are three events $e_1, e_2, e_3$ in which Al dug a hole, Bill inserted a rosebush in it, and Carl covered the rosebush with soil. Then there is also an event $e_4$ in which Al, Bill, and Carl planted a rosebush. Let $e_4$ be this event. If $e_4 = e_1 \oplus e_2 \oplus e_3$, we have a counterexample to (2).

Exercise 3.1 Why is this a counterexample? How could one respond to this challenge? □

3.3 Telicity, quantization, minimal parts

• Predicates can be telic or atelic.

– Atelic predicates: walk, sleep, talk, eat apples, run, run towards the store
  ($\approx$ as soon as you start X-ing, you have already X-ed)
– Telic predicates: build a house, finish talking, eat ten apples, run to the store
  ($\approx$ you need to reach a set terminal point in order to have X-ed)

• Traditionally, atelicity is understood as the subinterval property or divisive reference. Telicity is understood as quantized reference. This brings out the parallel between the telic/atelic and count/mass oppositions (e.g. Bach 1986).

(3) a. telic : atelic :: count : mass
   b. quantized : (approximate) subinterval :: quantized : (approximate) divisive

• My dissertation (Champollion 2010) suggests a generalized notion that encompasses both the subinterval property and divisive reference.

• We will use the following definition of the subinterval property:
(4) \( \text{SUBINTERVAL}(P) =_{\text{def}} \forall e[P(e) \rightarrow \forall i[i < \tau(e) \rightarrow \exists e'[P(e') \land e' < e \land i = \tau(e')]]] \)
(Whenever P holds of an event e, then at every subinterval of the runtime of e, there is a subevent of which P also holds.)

• for-adverbials require atelicity because they presuppose the subinterval property:

(5) *eat ten apples for three hours
Failing presupposition: \( \text{SUBINTERVAL}([\text{eat ten apples}]) \), i.e. every part of the runtime of an eating-ten-apples event e is the runtime of another eating-ten-apples event that is a part of e.

• The “minimal-parts problem” (Taylor 1977; Dowty 1979): The subinterval property distributes P literally over all subintervals. This is too strong.

(6) John and Mary waltzed for an hour
\( \not\Rightarrow \) #John and Mary waltzed within every single moment of the hour
\( \Rightarrow \) John and Mary waltzed within every short subinterval of the hour

• The length interval that counts as very small for the purpose of the for-adverbial varies relative to the length of the bigger interval:

(7) The Chinese people have created abundant folk arts … passed on from generation to generation for thousands of years.¹

• The subinterval property can be relaxed to account for this problem (Champollion 2010)

3.4 Aspectual composition

• Aspectual composition is the problem of how complex constituents acquire the telic/atelic distinction from their parts (Verkuyl 1972; Krifka 1998)

• With “incremental theme” verbs like eat, the correspondence is clear:

(8) a. eat apples / applesauce for an hour
b. *eat an apple / two apples / the apple for an hour

(9)  
  a. count : mass :: telic : atelic  
  b. apple : apples :: eat an apple : eat apples

(10)  
  a. drink wine for an hour  
  b. *drink a glass of wine for an hour

• With “holistic theme” verbs like *push and see, the pattern is different:

(11)  
  a. push carts for an hour  
  b. push a cart for an hour

(12)  
  a. look at apples / applesauce for an hour  
  b. look at an apple / two apples / the apple for an hour

• Verkuyl’s Generalization (Verkuyl 1972): When the direct object of an incremental-theme verb is a count expression, we have a telic predicate, otherwise an atelic one.

• Krifka (1992): in incremental-theme verbs (also called “measuring-out” verbs among other things), the parts of the event can be related to the parts of the theme (see Figure 3.1).

  Figure 3.1: Incremental theme of *drink wine, from Krifka (1992)

• We can formalize holistic and incremental themes via meaning postulates (Krifka 1998; this is a simplified version)

(13)  
  Definition: Incrementality  
  \[ \text{Incremental}_\theta(P) \iff \forall e \forall e' \forall x [\theta(e) = x \land e' < e \rightarrow \theta(e') < x] \]
Definition: Holism
Holistic_θ(P) ⇔ ∀e∀e′∀x [θ(e) = x ∧ e′ < e → θ(e′) = x]

Meaning postulates
a. Incremental_theme([eat])
   b. Incremental_theme([drink])
   c. Holistic_theme([see])

• Then we apply these meaning postulates to prove or disprove that the various VPs above
  have divisive reference or the subinterval property.

• Claim: [eat two apples] does not have the subinterval property.

• Proof: Suppose it has, then let e be an event in its denotation whose runtime is an hour.
  From the definition of the subinterval property, (4), at each subinterval of this hour there
  must be a proper subevent of e whose theme is again two apples. Let e′ be any of these
  proper subevents. Let the theme of e be x and the theme of e′ be y. Then x and y are each a
  sum of two apples. From the “incremental theme” meaning postulate in (15a) we know that
  y is a proper part of x′. Since two apples is quantized, x and y can not both be two apples.
  Contradiction.

Exercise 3.2 Why does the proof not go through for see two apples? Why does it not go through
for eat apples? □
Appendix

**Answer to Exercise 2.1:** There are potential counterexamples. A twig may have a part that is again a twig, a rock may have a part that is again a rock, and so on (Zucchi and White 2001). Similarly for nouns like fence and wall (Rothstein 2004). For these cases, one can rescue the idea that all nouns have quantized reference by assuming that context specifies an individuation scheme (Chierchia 2010; Rothstein 2010).

**Answer to Exercise 2.2:** The claim that all mass terms have divisive reference is at best true as an approximation because of the minimal-parts problem (pea soup – a pea is not pea soup, fruit cake – a sultana is not fruit cake, succotash – consists of corn and beans), etc.)

**Answer to Exercise 3.1:** If we consider \( e_4 = e_1 \oplus e_2 \oplus e_3 \), we have a counterexample for the following reasons. The themes of \( e_1, e_2, e_3 \) are the hole, the rosebush, and the soil, while the theme of \( e_4 \) is just the rosebush. The theme of \( e_4 \) is not the sum of the themes of \( e_1, e_2, \) and \( e_3 \). This violates cumulativity.

One way to respond to this challenge is to reject the assumption that the mereological parthood relation should model all parthood relations that can be intuitively posited (see Section 1.2). In this case, we do not need to assume that \( e_4 \) is actually the sum of \( e_1, e_2, \) and \( e_3 \). Even though the existence of \( e_4 \) can be traced back to the occurrence of \( e_1, e_2, \) and \( e_3, \) nothing forces us to assume that these three events are actually parts of \( e_4 \), just like we do not consider a plume of smoke to be part of the fire from which it comes, even though its existence can be traced back to the fire. Without the assumption that \( e_4 \) contains \( e_1 \) through \( e_3 \) as parts, Kratzer’s objection against cumulativity vanishes. See also Williams (2009) and Piñón (2011) for more discussion.

**Answer to Exercise 3.2:** For see two apples, the proof does not go through because the theme of see is holistic and not incremental, that is, there is no meaning postulate like Incremental\(_{\text{theme}}\)(see\). For eat apples, the proof does not go through because apples is not quantized (the sum of any two things in the denotation of apples is again in the denotation of apples).
Bibliography


