Drawing Finite State Automata
Answers to Exercise 1.4 from Sipser (p84)
Ling 106
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Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is \(\{0,1\}\).

1. \(\{w | w \text{ begins with a 1 and ends with a 0}\}\).

\[ \]

2. \(\{w | w \text{ contains at least three 1s}\}\).

\[ \]

3. \(\{w | w \text{ contains the substring 0101, i.e., } w = x0101y \text{ for some } x \text{ and } y\}\).

\[ \]

4. \(\{w | w \text{ has length at least 3 and its third symbol is a 0}\}\).

\[ \]
5. \{w \mid w \text{ starts with 0 and has odd length, or starts with 1 and has even length}\}.

6. \{w \mid w \text{ doesn’t contain the substring 110}\}.

7. \{w \mid \text{the length of } w \text{ is at most 5}\}.

8. \{w \mid w \text{ is any string except 11 and 111}\}.
9. \( \{ w \mid \text{every odd position of } w \text{ is } 1 \} \).

\[
\begin{tikzpicture}
  \node[state] (q0) at (0,0) {$q_0$};
  \node[state] (q1) at (1,1) {$q_1$};
  \node[state] (q2) at (1,-1) {$q_2$};
  \draw[->] (q0) edge node[above] {1} (q1);
  \draw[->] (q0) edge node[below] {0,1} (q2);
  \draw[->] (q1) edge node[below] {0,1} (q2);
\end{tikzpicture}
\]

10. \( \{ w \mid w \text{ contains at least two } 0s \text{ and at most one } 1 \} \).

\[
\begin{tikzpicture}
  \node[state] (q0) at (0,0) {$q_0$};
  \node[state] (q1) at (1,1) {$q_1$};
  \node[state] (q2) at (1,-1) {$q_2$};
  \node[state] (q3) at (-1,1) {$q_3$};
  \node[state] (q4) at (-1,-1) {$q_4$};
  \node[state] (q5) at (2,0) {$q_5$};
  \node[state] (q6) at (3,1) {$q_6$};
  \draw[->] (q0) edge node[above] {1} (q1);
  \draw[->] (q0) edge node[below] {0} (q3);
  \draw[->] (q0) edge node[above] {1} (q5);
  \draw[->] (q1) edge node[below] {1} (q4);
  \draw[->] (q1) edge node[above] {0} (q5);
  \draw[->] (q2) edge node[below] {1} (q4);
  \draw[->] (q2) edge node[above] {1} (q5);
  \draw[->] (q3) edge node[below] {1} (q5);
  \draw[->] (q3) edge node[above] {0} (q4);
  \draw[->] (q4) edge node[below] {1} (q5);
  \draw[->] (q4) edge node[above] {0} (q3);
\end{tikzpicture}
\]

11. \( \{ \epsilon,0 \} \).

\[
\begin{tikzpicture}
  \node[state] (q0) at (0,0) {$q_0$};
  \node[state] (q1) at (1,1) {$q_1$};
  \node[state] (q2) at (1,-1) {$q_2$};
  \node[state] (q3) at (-1,1) {$q_3$};
  \draw[->] (q0) edge node[above] {0,1} (q1);
  \draw[->] (q0) edge node[below] {0,1} (q3);
  \draw[->] (q1) edge node[below] {0,1} (q2);
\end{tikzpicture}
\]
12. \( \{w \mid w \text{ contains an even number of 0s, or exactly two 1s} \} \).

13. The empty set.

14. All strings except the empty string.