COGS501 Placement Test

Sept. 6, 2007

Your name:

Department and year:

Email address:

Circle the best description of your involvement in this course:

- For Credit
- Regular auditor
- Not sure yet

Briefly sketch your background in

1. Mathematics

2. Computer programming (any languages)
1. \((x + 1) \times (x - 2) = \)
   
   a) \(2x - 1\)
   b) \(x^2 - 2\)
   c) \(x^2 - x - 2\)
   d) \(x^2 + x - 2\)
   e) \(x^2 - 3x - 1\)

2. The distance from \((0, 0)\) to \((x, y)\) is
   
   a) \(x + y\)
   b) \(xy\)
   c) \(x^2 + y^2\)
   d) \((x^2 + y^2)^{1/2}\)
   e) \(\log_2(2x + 2y)\)

3. Two vectors are orthogonal iff
   
   a) their inner product is one.
   b) their inner product is zero.
   c) their sum is one.
   d) their sum is zero.
   e) (none of the above)

4. If a function \(f(x)\) is linear, then (circle all that apply):
   
   a) \(f(cx) = cf(x)\)
   b) \(f(c + x) = c + f(x)\)
   c) \(f(x + y) = f(x) + f(y)\)
   d) \(f(xy) = f(x)f(y)\)

   Note: \(x\) and \(y\) are variables; \(c\) is a constant.
5. If $A$ is a matrix \( \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \) then $A^2$ is

a) 29  

b) \( \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix} \)  

c) \( \begin{pmatrix} 1 & 4 \\ 9 & 16 \end{pmatrix} \)  

d) \( \begin{pmatrix} 1 & 2 & 1 & 2 \\ 3 & 4 & 3 & 4 \end{pmatrix} \) 

6. Write down the following products of a matrix times a vector. Circle each of the given vectors (one or more) that is an eigenvector of the given matrix.

a) \( \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} \)

b) \( \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} \)

c) \( \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \)

d) \( \begin{pmatrix} 4 & -5 \\ 2 & -3 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \)

7. The null space of a 3 x 3 matrix

a) is necessarily empty  

b) has at least three dimensions  

c) is the image of an ellipse under multiplication by the matrix  

d) is invariant under multiplication by the matrix  

e) is mapped to 0 by the matrix
8. The principal component of a multi-dimensional data set
   a) gives the direction of greatest variance
   b) is the grand mean
   c) gives the direction of least variance
   d) contains most of the data
   e) has the largest marginal sum

9. The product of the complex numbers $2 + 3i$ and $3 + 2i$ is
   a) 13
   b) $-1 + i$
   c) $6 + 6i$
   d) $5 + 5i$
   e) $13i$

10. $\sin^2(x) + \cos^2(x) =$
    a) 1
    b) $e^x$
    c) $\pi$
    d) 0
    e) e

11. A gaussian distribution
    a) is uniform
    b) has minimum entropy
    c) always has zero mean
    d) never has zero mean
    e) is maximal at its mean
12. In a normal distribution the proportion of samples within one standard deviation (on either side) of the mean is about

a) 25%  
b) 1%  
c) 100%  
d) 57%  
e) 68%

13. If \( \{p_i\} \) is a discrete probability distribution then \( \sum_i p_i = \)

a) 0  
b) 1  
c) \( 1/\pi \)  
d) \( \pi \)  
e) \( P \)

14. If \( X \) and \( Y \) are independent \( P(X|Y) = \)

a) 0  
b) 1  
c) \( P(X) \)  
d) \( P(Y) \)  
e) \( P(Y|X) \)
15. Bayes' Theorem says that

a) \( P(XY) = P(X)P(Y) \)
b) \( P(X|Y) = P(Y|X) \)
c) \( P(X|Y) = P(Y|X)P(Y)/P(X) \)
d) \( P(X|Y) = P(Y|X)P(X)/P(Y) \)
e) \( P(XY) = P(X|Y)P(Y) \)

16. The variance of the set of numbers \{-1, 0, 1\} is

a) \(-1\)
b) \(0\)
c) \(1\)
d) \(\sqrt{2}\)
e) \(2\)

17. You want to test the hypothesis that, in the dialect of a particular region, the vowel in the word “tied” is longer than that in the word “tide”. You have obtained samples of speech from speakers from that region and measured the lengths of examples of the vowels from the two words, obtaining means and standard deviations. To test whether the means are significantly different you use

a) a chi-squared test
b) analysis of covariance
c) linear discriminant analysis
d) a t test
e) the Bonferroni correction
18. The Nyquist frequency
   a) is half the sampling frequency
   b) is twice the sampling frequency
   c) is 10 times the log of the sampling frequency
   d) is the square of the sampling frequency
   e) is the square root of the sampling frequency

19. A sound has a wavelength of 2ft. Taking the speed of sound as 1128ft/second, the frequency of the sound is
   a) $\frac{2}{1128}$ Hz
   b) $\frac{1128}{2}$ Hz
   c) $1128 \times 2$ Hz
   d) $\sqrt{1128 \times 2}$ Hz
   e) $(1128 \times 2)^2$ Hz

20. Which of these functions is its own Fourier transform?
   a) an impulse
   b) a sine wave
   c) a gaussian
   d) a decaying exponential
   e) a step function