# Bayesian Decision Theory, Iterated Learning and Portuguese Clitics

## Psychocomputational Models of Human Language Acquisition

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# Computational Models of Language Change

*Some questions:*

- How can we deal with individual and population variation in models of language change?
- Where does instability come from in these models?
- How do we use all these frequency counts to choose a grammar?

*Some Frameworks:*

- Iterated learning: (Kirby, 2001; Kirby et al., 2007)
- Dynamical systems: (Mitchener and Nowak, 2003; Nowak et al., 1999).
- Social learning: (Niyogi and Berwick, 1998; Yang, 2001)

# How do you make a decision?

*The decision rule through which a grammar is selected is crucial!*

- Are learners *just* trying to fit the probability distribution of the input data to a predefined model? This is basically what Maximum Likelihood Estimation (MLE) allows you to do.
- MLE requires us to conflate several factors: innate biases (priors), social and communicative factors, and random noise.
- If we view language learning as a problem involving *beliefs* and factors outside pure point estimation, the Bayesian view becomes very attractive.
- However, even within general Bayesian frameworks, MLE is still often implicitly employed (cf. MAP estimation Griffiths and Kalish (2005); Dowman et al. (2006); Briscoe (2000))

What does it mean to be a Bayesian?

# Outline

- Portuguese clitic data
- Models and requirements
- Bayesian decision theory

# Portuguese Direct Object Clitics

(1) $a_1$ Paolo a ama (affirmative, proclisis)

$a_2$ Paolo ama-a (affirmative, enclisis)

$a_3$ Quem a ama (obligatory proclisis)

- Affirmative sentences with topics, adjuncts or referential subjects:
  - Classical Portuguese (ClP, 16th to mid-19th century): allowed direct object enclitics and proclitics (preferred Galves et al. (2005a)). ($a_1$, $a_2$)
  - Modern European Portuguese (EP): obligatory enclisis. ($a_2$)
- Proclitic forms are obligatory in other syntactic contexts. ($a_3$)

*Note: we're treating EP as a subset of ClP.*

# Change!

- According to corpus studies (Galves et al., 2005a), there was a sharp rise in enclisis in the early to mid 18th century.
- Galves and Galves (1995): this syntactic change was driven by change in stress patterns. (although see Galves (2003); Costa and Duarte (2002)).
- *Acquisition question:* How does the learner do parameter setting?
- *Production question:* What sort of data will the learner produce for the next generation?

# The Galves Batch Model

- Galves and Galves (1995): Construction types are given a probability proportional to the stress contour associated with it.
- Clauses of type $a_1$ and $a_3$ (proclisis) have weight $p$ and clauses of type $a_2$ (enclisis) have weight $q$.

$$\mathbf{P}(a_1|G_{CIP}) = p/(2p+q) \tag{2}$$
$$\mathbf{P}(a_1|G_{EP}) = 0 \tag{3}$$

- Grammar selection via Maximum Likelihood Estimation (MLE).
- The probability of the learner acquiring $G_{CIP}$ as the probability that clause type 1 occurs at least once in $n$ samples (the critical period).

# Batch Learning as Markov Process

- Niyogi and Berwick (1998); Niyogi (2006): re-implement the GBM but take more of a a population level view.
- $\alpha_t$ = proportion of the population with $G_{EP}$ at time $t$.
- $\alpha_{t+1}$ depends on $\alpha_t$ and the learning mechanism (MLE). (Markov process with two states)

$$
\begin{aligned}
\mathbf{P}(a_1|G_{CIP}) = \mathbf{P}(a_3|G_{CIP}) &= p \text{ for some } p \in [0,1], \\
\mathbf{P}(a_2|G_{CIP}) &= 1 - 2p. \\
\mathbf{P}(a_1|G_{EP}) &= 0, \\
\mathbf{P}(a_2|G_{EP}) &= q, \text{ for some } q \in [0,1], \\
\mathbf{P}(a_3|G_{EP}) &= 1 - q,
\end{aligned}
$$

- $p$ and $q$ are *production probabilities* encoded in the grammar. These hold across the board for *all* speakers of a particular grammar.

# And so...

- Learners may still acquire $G_{CIP}$ even though they do not see any instances of variational proclisis ($a_1$)! That is, if there are *too many* instances of the type $a_3$ (obligatory proclisis).
- Is because there is proclisis in $a_3$? No! This would still happen if the syntax of the $a_3$ type was totally devoid of clitics.
- Also, a learner who acquires $G_{CIP}$ will continue to use a (possibly) very high rate of variational proclisis ($p$) in spite of being surrounded by $G_{EP}$ speakers.
- Shouldn't we expect that the desire to communicate would pressure speakers of $G_{CIP}$ to lower the rate of variational proclisis in the face of multitudes of $G_{EP}$ speakers?
- How do we deal with noise? (c.f. Briscoe (2002)) What about biases? How about being Bayesian?

# Bayesian Iterated Learning

Signal/meaning pairs (Griffiths and Kalish, 2005)

- $(Y_k, X_k) = \{(y_1, x_1) \dots (y_n, x_n)\}$: (utterance, meaning) pairs received by agent in generation $k$. ($y \rightarrow x$ is many to one).
- This allows us to focus only on types that show variation.
- Grammar selection is based on the posterior ($g$ is the hypothesized grammar),

$$\mathbf{P}(g|X_k, Y_k) = \frac{\mathbf{P}(Y_k|X_k, g)\mathbf{P}(g)}{\mathbf{P}(Y_k|X_k)},$$

  Priors over grammars are assumed to be innate and invariable across generations.
- Also, add an error term to account for random noise.
- Griffiths and Kalish (2005); Kirby et al. (2007) show analytically that convergence to the prior depends on the selection mechanism (MAP, sampling from the posterior, etc.)

# BIL and Portuguese

- BIL $\approx$ Griffiths and Kalish (2005) does not take into account variation in the community (!). However, in general IL allows more than one agent in a generation Kirby and Hurford (2002).
  $\Rightarrow$ BIL is like the previous models except for the priors.
- For Portuguese, we don't have to consider cases of obligatory proclisis ($a_3$) since they do not differentiate the two grammars.
- However, $\mathbf{P}(a_1|x_1) = p$ is still seems to be an innate part of the grammar with MAP estimation.

# What would we like in the model?

- Frameworks are frameworks – they still need articulation.
- We would like to incorporate some formal notion of why frequency estimation is important to the learner.
- At least part of this should come from the fact that the learner wishes to communicate effectively with a variety of speakers.
- For example, we want to incorporate the intuitive idea that using rare forms when frequent forms exists may be disfavored.
- Also forms that are harder to produce (and process) should be disfavoured (c.f. the prosody argument).
- The decision problem that learners face is subjective – learners choose a grammar that they believe will be most useful for them. That is, they make decisions based on *expected utility*.

# The Components of the Bayesian Decision Rule

Bayesian decision rule: maximize the expected utility of taking: action $a$ from decision set $\Theta$ with respect to the possible values of $\theta$ and the observed values of $y$. That is,

$$\hat{a} = \text{argmax}_a \int_\Theta U(a, \theta)\mathbf{P}(\theta|y)d\theta$$

- The likelihood function
- The prior
- The utility function
- The decision rule
- The production distribution

# The Parameter setting problem

- Things the learner doesn't know but would like to (parameters, $\theta$):
  - $\alpha =$ proportion of $G_{EP}$ speakers [syntactic parameter 'ON']
  - $p =$ rate of enclisis of $G_{CIP}$ speakers
- The only evidence the learner has for any given parameter is the count of inputs that support the parameter setting and a count of those that oppose it (observations, $y$).
- The task of the learner is to use these frequency counts to evaluate what is the best grammar for them (decision set, $\Theta$).

# The Likelihood Function

- Treat the data as $N$ (independent) Bernoulli trials: $S_N = \{(y_i, x_i)\}_{i=1}^{N}$.
- Let, $k$ be the number of cases that were parseable with parameter setting on. e.g. enclitics.
- The likelihood function is:

$$\mathbf{P}(S_N|\alpha, p) = \binom{N}{k}[(1-\alpha)p+\alpha]^k[(1-\alpha)(1-p)]^{N-k}$$

- $\alpha, p$ are dummies here, they aren't part of the grammar.
- **Note:** $G_{EP}$ is a subset of $G_{CIP}$ so does not present any counter-evidence for $G_{CIP}$ in this model.

# The Prior

Prior beliefs of the learner about possible combinations of $\alpha$ and $p$:

- If $\alpha = 1$ then the population is entirely made up of $G_{EP}$ speakers, the value of $p$ is irrelevant as it only applies to $G_{CIP}$ speakers.
- The simplest hypothesis is that $\alpha = 1$, $p = 1$ is a maximum. i.e. before being wiped out, $G_{CIP}$ speakers would have increasingly used enclitic constructions to fit with the rest of the population.
- Similarly, if $\alpha = 0$ then the population would most likely be using proclitic construction a large proportion of the time. So, maxima around $\alpha = 0$, $p = 0.05$ (Galves et al., 2005b).

# The Prior

As a function:

$$f(\alpha, p) = \frac{1}{c} e^{-(p-(0.95\alpha+0.05))^2}$$

where $c$ is a normalizing constant. $f(\alpha, p)$ is then just a squared Gaussian with mean $0.95\alpha + 0.05$. This is the rate of enclisis found in the Tycho Brahe corpus.

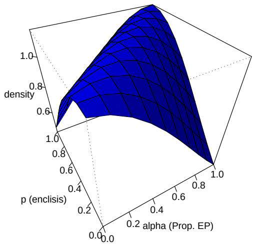



Figure: The prior density: $f(\alpha, p)$

# The Utility Function

- The learner wants to acquire the same grammar as the rest of its community.
- The learner wants to be able to play both roles of speaker and hearer successfully.
- A speaker of CIP will be able to understand both EP and CIP speakers without any penalty. However, a speaker of EP will have difficulty understanding CIP speakers. Conversely, EP speakers will be able to converse without penalty but not vice-versa.
- Assume the individual plays speaker/hearer half the time.

$$U(a, \alpha, p) = \begin{cases} -\frac{1}{2}\alpha & \text{if } a = 0 \ (G_{CIP}), \\ -\frac{1}{2}(1-\alpha) & \text{if } a = 1 \ (G_{EP}). \end{cases}$$

*This is also where we should be encoding pronounciation difficulty!*

# Utility Maximization

The learner does not actually know what $\alpha$ and $p$ are. They need to *infer* it from frequencies $k$ and $N$. Instead of trying to pin this down (or stipulate it) expected utility maximization hedges its bets. So,

$$\mathbf{E}[U(a,\alpha,p)|S_N] = \int_{[0,1]^2} U(a,\alpha,p)d\mathbf{P}(\alpha,p|S_N).$$

To find out whether the parameter should be set 'off' (and simplified), we calculate:

$$\mathbf{E}[U(0,\alpha,p)|S_N] > \mathbf{E}[U(1,\alpha,p)|S_N].$$

$$\int_{[0,1]^2} (2\alpha - 1)\mathbf{P}(S_N|\alpha,p)f(\alpha,p)d(\alpha,p) < 0$$

If this last statement is true, the learner should choose $G_{CIP}$.

# Estimating Production Rates

- Assume that production probabilities are derivable from the frequencies observed in the acquisition process.
- For a CIP speaker:

$$\mathbf{P}(a_1|x_1) = (N-k)/N,$$
$$\mathbf{P}(a_2|x_1) = k/N$$

- For an EP speaker:

$$\mathbf{P}(a_2|x_1) = 1.$$

- Let $\alpha_0$ be the proportion of $G_{EP}$ speakers observed in generation 0. Then the probability of getting the enclitic version in the first round.

$$q_0 = \mathbf{P}_{pop}(a_2|x_1, T=0) = (1-\alpha_0)p_0 + \alpha_0$$

# Over and Over...

- The proportion of speakers who will see $k$ enclitic constructions in $N$ Bernoulli trials is:

$$\binom{N}{k} q_t^c (1 - q_t)^{N-k}$$

  where $q_t$ is probability of seeing an enclitic in generation $t$.
- This proportion of speakers will then contribute enclitics with a rate of $k/N$ to the next generation, $t + 1$.

# Initial $G_{CIP}$ rate of enclisis between 60-70%

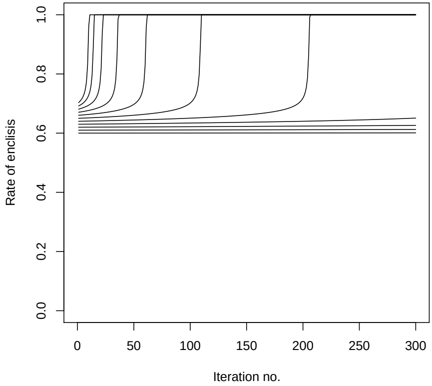



Figure: Rate of enclisis. $N = 100$, $\alpha_0 = 0$, and the initial rate of $G_{CIP}$ enclisis, $p$, ranges from 0.6 to 0.7.

*Change doesn't take off from* $p = 0.05$*!*

# More interactions?

- The stability of $G_{CIP}$ is really assumed by the model via the prior.
- Crucially, the simulation above still does not incorporate the effects of simultaneous change in other modules of language (e.g. phonology).
- Production changes? We could define a new decision problem that estimates production probabilities.

# Conclusion

Take home points:

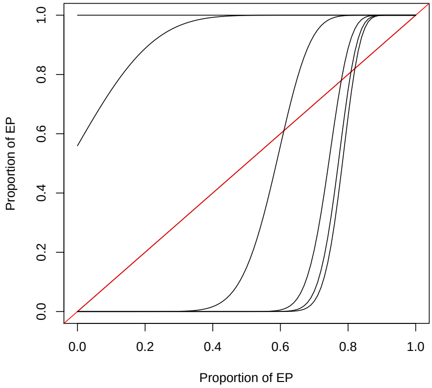
- This model articulates the general social learning model Niyogi (2006): learners learn from an (infinite) population.
- The decision procedure was presented as a utility maximizing decision rule where the learner estimates population frequencies in order to maximize communicability.
- Ideally we would look at a change in progress where we could do better estimation of the prior and utility functions.

*Thanks!*

Especially to: Charles Yang, Andrew Clausen, and Ling 575/506-ers!

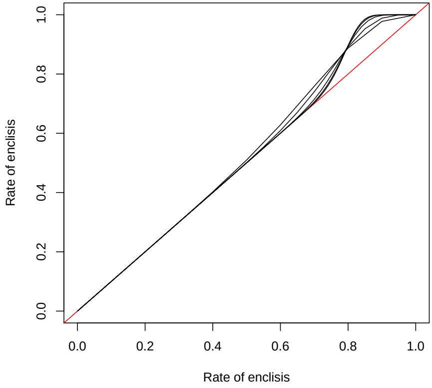
# Simulation: 300 iterations

Figure: Transition diagram: Proportion of $G_{EP}$ speakers. Different curves represent different $G_{CIP}$ enclisis rates ($p$): 0.05, 0.1, 0.2, 0.5, 0.8, and 1. $n = 100$ and $\alpha = 0$.



# Different input sizes

Figure: Transition diagram: Overall rates of enclisis. Different curves represent different input sizes *n*: $n = 10, 20, 50, 80, 100$. $\alpha = 0$



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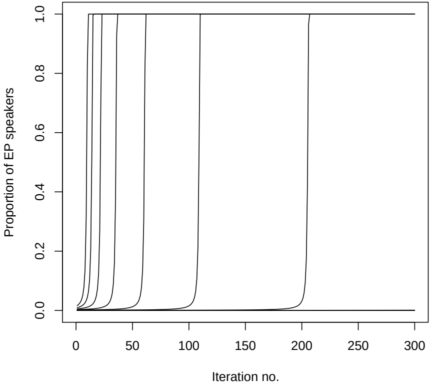


Figure: Proportions of $G_{EP}$ speakers. $n = 100$, $\alpha_0 = 0$, and the initial rate of $G_{CIP}$ enclisis, $p$, ranges from 0.6 to 0.7.

# Portuguese BIL Example

- If there is a 1-1 mapping between a meaning $x$ and a type $y$ then

$$\mathbf{P}(y|x, G) = 1 - \epsilon,$$

  if $G$ admits $x$ ($\epsilon$ is an error term).
- Let frequencies for input be: $a_1 = a, a_2 = b, a_3 = c$.
- Let, $x_1, x_3$ be the meanings associate with $a_1$ and $a_3$ respectively.
- We do not need to consider the contribution of obligatory proclisis to calculate the MLE (or MAP) grammar.

$$
\begin{aligned}
\mathbf{P}(G|Y_k, X_k) &\propto \mathbf{P}(Y_k|X_k, G)\mathbf{P}(G) \\
&= \prod_{i=1}^{k} \mathbf{P}(y_i|x_i, G)\mathbf{P}(x_i) \\
&= \mathbf{P}(a_1)^a \mathbf{P}(a_2)^b \mathbf{P}(a_3)^c \;\; \mathbf{P}(a_1|x_1, G)^{a'} \mathbf{P}(a_1|x_3, G)^{a''} \\
&\quad \mathbf{P}(a_2|x_1, G)^{b'} \mathbf{P}(a_2|x_3, G)^{b''} \\
&\quad \mathbf{P}(a_3|x_1, G)^{c'} \mathbf{P}(a_3|x_3, G)^{c''} \mathbf{P}(G)
\end{aligned}
$$

Where $a = a' + a''$ and similarly for the other frequency counts. Proclisis in affirmative sentences is simply given the error probability, $\epsilon$, in $G_{EP}$.

If we only care about finding MLE (or MAP) grammar, and taking probabilities from Nigoyi's implementation of GBM, then we have the following.

$$\mathbf{P}(G_{CIP}|Y_k, X_k) \propto \mathbf{P}(G_{CIP}) \frac{(p - \epsilon/2)}{(1 - p - \epsilon))^{a'}} \frac{((1 - 2p - \epsilon/2)}{(1 - p - \epsilon)^{b'}}$$

$$\mathbf{P}(G_{EP}|Y_k, X_k) \propto \mathbf{P}(G_{EP})(\epsilon/2)^{a'}(1 - \epsilon/2)^{b'}$$

- The explicit connection between meaning and types allows us to reduce the parameter space needed to evaluate the two grammars in question.
- We only need to parameterize the error term to do the the likelihood computation for $G_{EP}$. In general, it will allow us to focus only on types that show variation.
- The prior notwithstanding, this reduction in the parameter space is welcome in comparison with Nigoyi's implementation.
- However, this still suffers from over-parameterization the problems associated with MLE. $\mathbf{P}(a_1|x_1) = p$ is still assumed to be an innate part of the grammar.

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