Problems in Population Models of Language Change

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University of York

Outline

- Frameworks for Population-Level Change
- Description of our Framework
- Population Size and Assumptions about the Grammar
- Generating S-Curves in Realistic Networks - the Cot-Caught Merger
- Capturing Complex Paths of Change - NCS in the St. Louis Corridor

Important Points

Population models and learning models interact

- **Assumptions must be carefully considered when modelling change**
- **Attested paths of change are governed by these interactions**
  - **Neither alone provides the full picture**
  - **Both should be studied to the extent possible**

Existing Frameworks

Three Classes of Framework

1. **Concrete Frameworks**
2. **Network Frameworks**
3. **Algebraic Frameworks**

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- **Individual agents on a grid moving randomly and interacting**
- **e.g., Harrison et al. 2002, Satterfield 2001, Schulze et al. 2008, Stanford & Kenny 2013**

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+ Gradient interaction probability for free
+ Diffusion is straightforward

- Not a lot of control over the network
- Thousands of degrees of freedom -> should run many many times -> slow
- Unclear how to include a learning model

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2. Network Frameworks
   - Speakers are nodes in a graph, edges are possibility of interaction
   - e.g., Baxter et al. 2006, Baxter et al. 2009, Blythe & Croft 2012, Fagyal et al. 2010, Minett & Wang 2008, Kauhanen 2016

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    \+ Much more control over network structure

    \+ Easy to model concepts from the sociolinguistic lit. (e.g., Milroy & Milroy)

    \- Nodes only interact with immediate neighbors -> slow and less realistic?

    \- Practically implemented as random interactions between neighbors -> same problem as #1

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3. **Algebraic Frameworks**
    - **Expected outcome of interactions in a perfectly mixed population is calculated analytically**
    - **Abrams & Stroganz 2003, Baxter et al. 2006, Minett & Wang 2008, Niyogi & Berwick 1997, Niyogi & Berwick 2009**

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   \+ Less reliance on random processes -> faster and more direct

   \+ Clear how to insert learning models into the framework

   \- No network structure! Always implemented over perfectly mixed populations

Our Framework

Best of Both Worlds

- An algebraic model operating on network graphs

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    - Models language change in social structures

Formal Description

Each iteration has two steps

1. **Diffusion - calculate how variants propagate**
2. **Transmission - calculate how variants are learned**

Diffusion

$$\mathbf{P}_{t+1} = \mathbf{B}^\top \alpha (\mathbf{I} - (1-\alpha)\mathbf{A})^{-1} \mathbf{H} (\mathbf{H}^\top \mathbf{H})^{-1}$$

- **A**   *n* x *n* adjacency matrix
- **α**   jump parameter
- **H**   *n* x *c* community-membership
- **B**   *c* x *g* distr. of grammars in comms
- **P**   *c* x *g* distr. of grammars in inputs

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The network graph

Who speaks what in what proportion

Who hears what in what proportion

Transmission

- **Dependent on the learning model**
- **Our implementation is modular, so many learning models can be slotted in**
    - **e.g., trigger-based learner (Gibson & Wexler 1994)**
    - **Variational learner (Yang 2000)**

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- Our implementation is modular, so many learning models can be slotted in
    - e.g., trigger-based learner (Gibson & Wexler 1994)
    - Variational learner (Yang 2000)
- Let L be the distribution of grammars internalized by a learner who heard P
    - L is a matrix consisting of g vectors l_1, l_2, … l_g
- Define *g* transition matrices T_1, T_2, … T_g, one for each potential target grammar

$$\mathbf{l}_i = \text{dominant eigenvector of} \sum_{j=1}^{g} \mathbf{P}_{t+1;j,i} \mathbf{T}_j$$

Transmission and Grammatical Advantage

- If L = P, learners internalize variants at the rate they hear them
    - This yields neutral change
- Otherwise, learners choose variants in a way that biases some over others
    - Some variants have an advantage over others
    - This yields S-curve change in perfectly mixed populations

Population Size and Grammars

Background

- Simulations typically run with a few hundred agents
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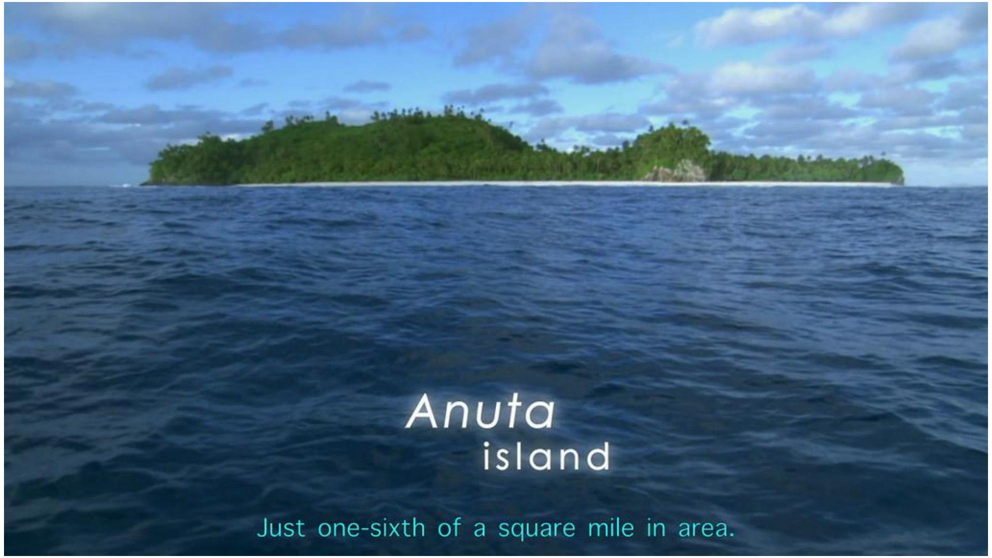

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- Do-Support (Ellegård 1953)
  - Rise of do-support constructions in English 1400-1700
  - Involved millions of individuals

When is this a Problem?

- **If learners internalize a distribution of grammars (i.e. competing grammars) *and* the population is (approximately) uniformly mixed, it is *not* a problem**
  - **Change closely approximates the path followed in infinite populations**
  - **So small-population models are a useful convenience**

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  - **Change closely approximates the path followed in infinite populations**
  - **So small-population models are a useful convenience**
- **But, if either of the above does not hold, it is a problem (maybe)**
  - **It becomes impossible to untangle population and learning effects**

Demonstration: Neutral Change

- **Assume two connected communities**
  - **C1 begins with 100% variant 1**
  - **C2 begins with 100% variant 2**

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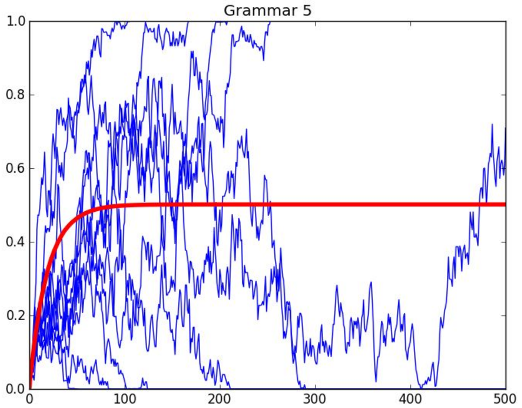
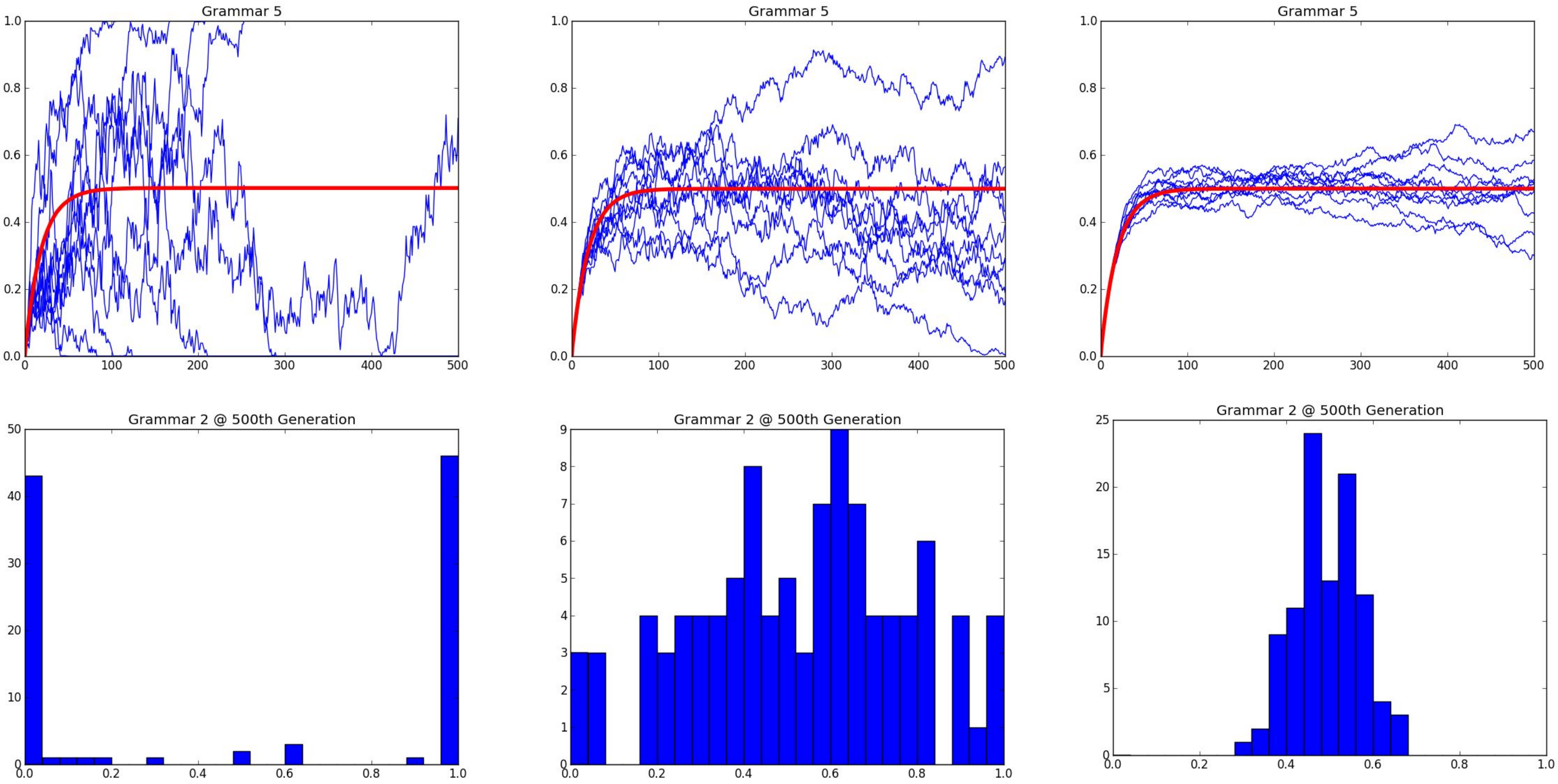
  - cf Kauhanen 2016

Rise of Variant 2 in C1
n = 200



Red curve predicted
Blue curves first 10 trials

n = 200 n = 2,000 n = 20,000



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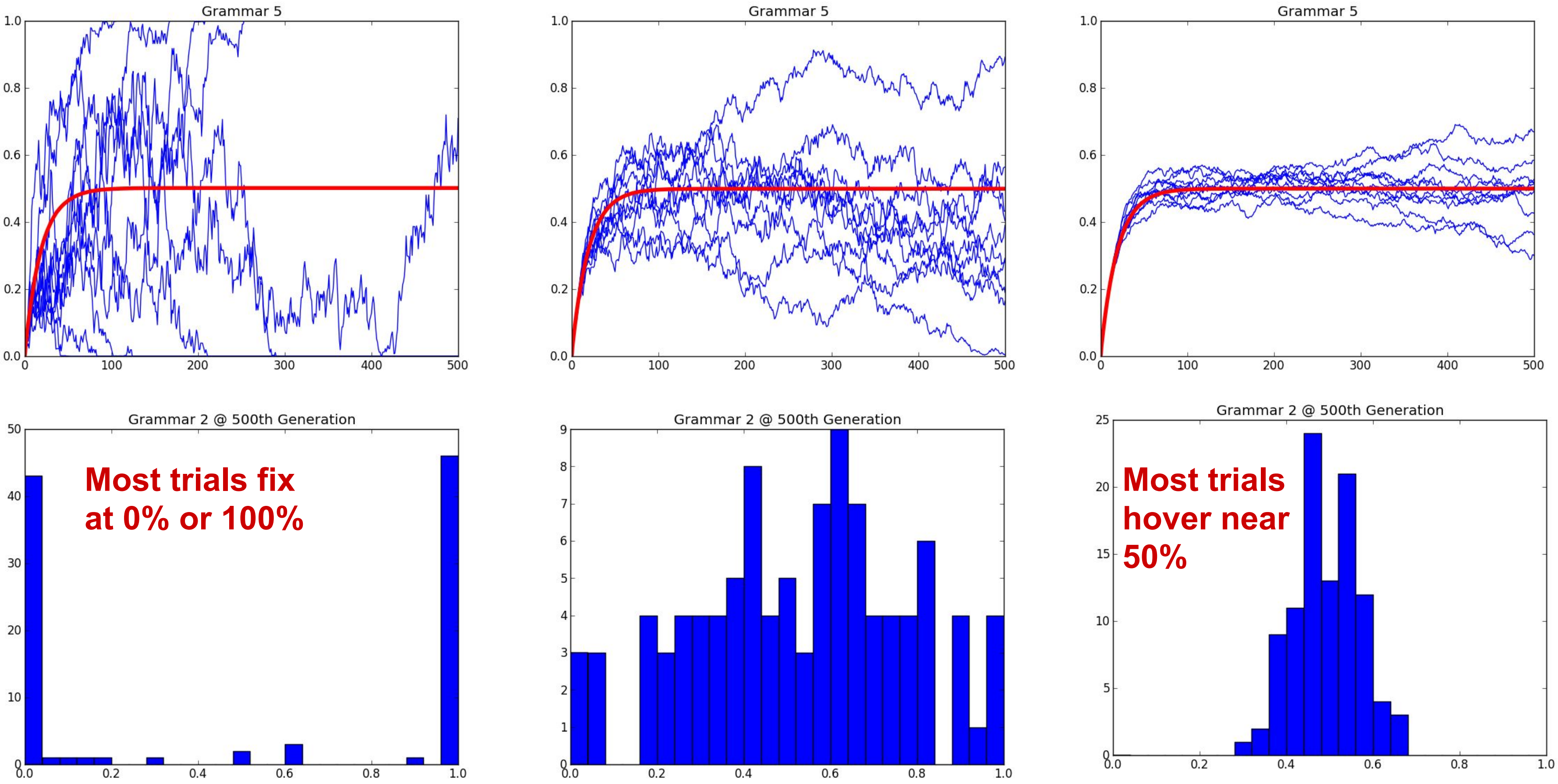

Demonstration: Advantage

- Repeating the previous test but with an advantage
  - Single community beginning at 1% innovative grammar
  - Learners choose a single grammar probabilistically, weighted toward innovative
  - Logistic curve predicted

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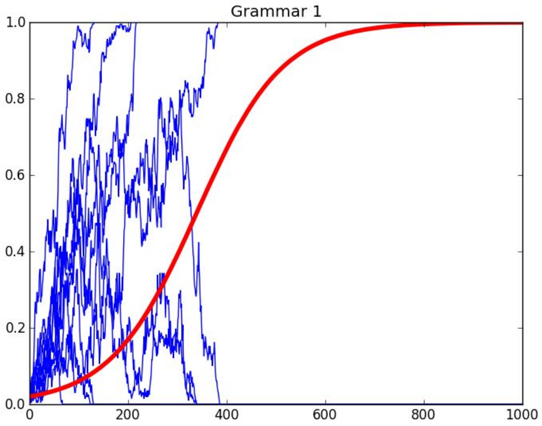

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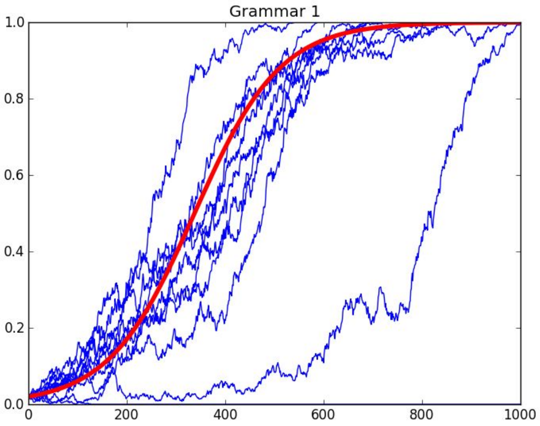

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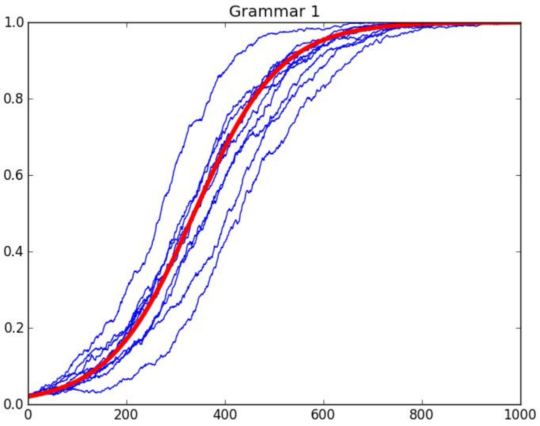

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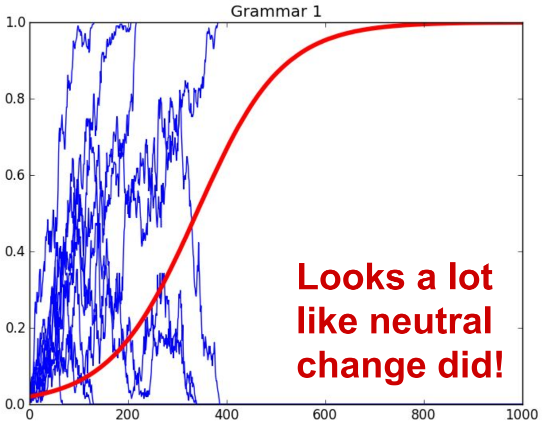

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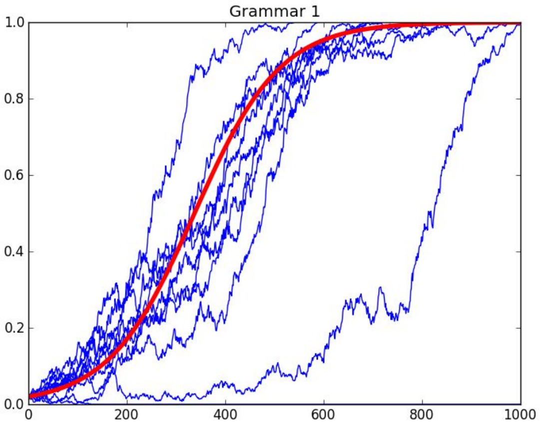

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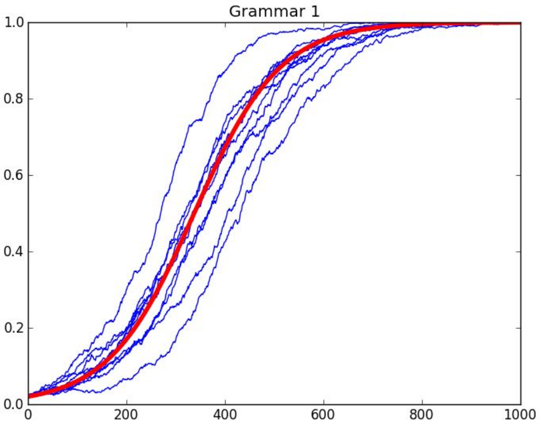

Demonstration: Advantage

- **At small *n*, S-curve change cannot arise**
- **At large n, S-curves become smooth**

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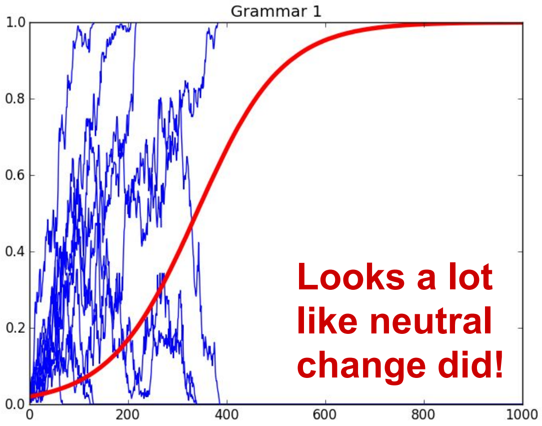

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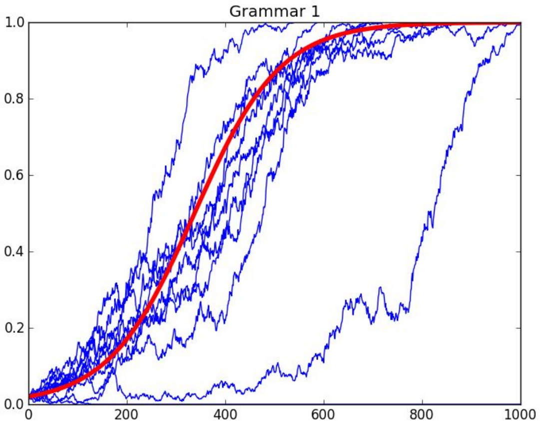

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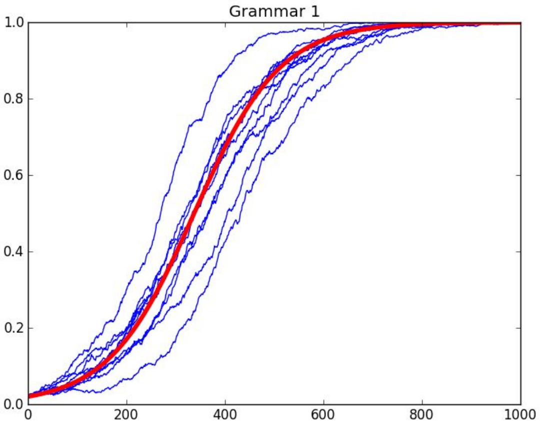

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    - Conclusions drawable for *n*=200 do not scale to *n*=20,000 or visa-versa

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- “Innocuous” assumptions may dominate behavior
  - Here, choice of population size and single-grammar assumptions
  - Conclusions drawable for n=200 do not scale to n=20,000 or visa-versa
- Slightly different assumptions yield drastically different conclusions
  - Is neutral change well-behaved?
  - Do we expect to see S-curve change?

Complex Networks and S-Curves:
The Cot-Caught Merger in New England

Single-Grammar Learners

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- **But it is not an indictment**
- **Some changes are neatly modeled as single-grammar processes**
  - **E.g., the spread of mergers, e.g., cot-caught on the RI/MA border (Johnson 2007, Yang 2009)**

The Cot-Caught Merger

- **/ɒ/ “cot” merges with /ɔ/ “caught”**
- **Usually unconditioned**

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  - Western PA
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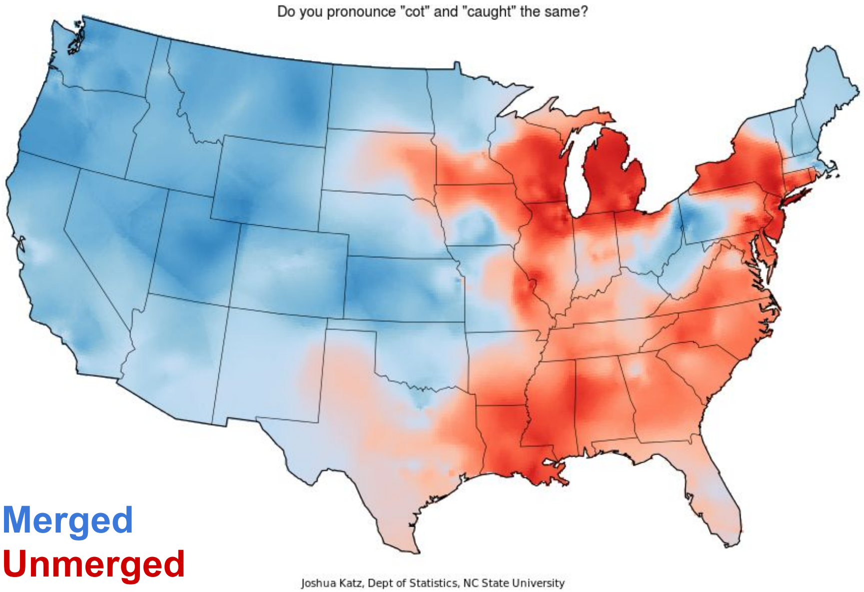
  - Canada (even NL!)



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Unmerged

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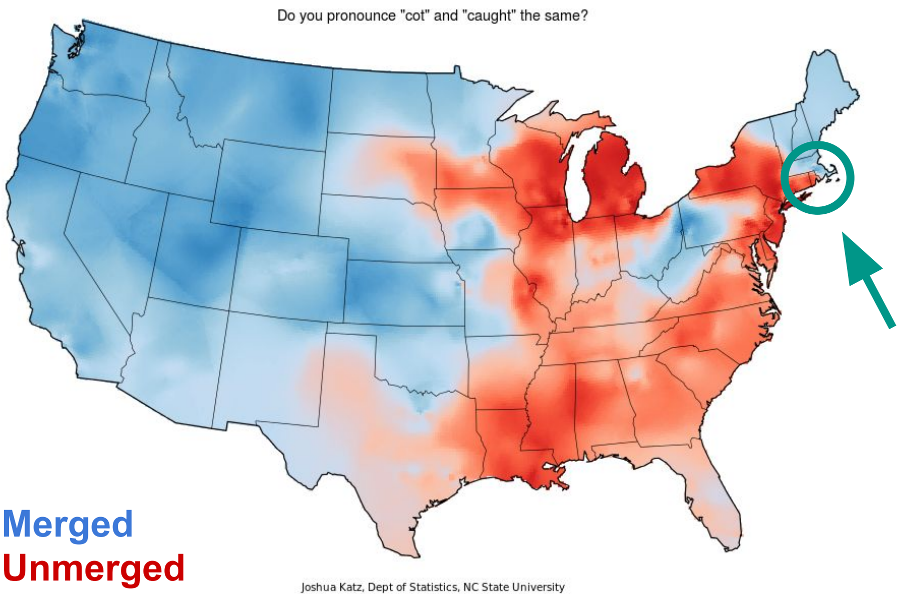
- It is spreading into Rhode Island (Johnson 2007)



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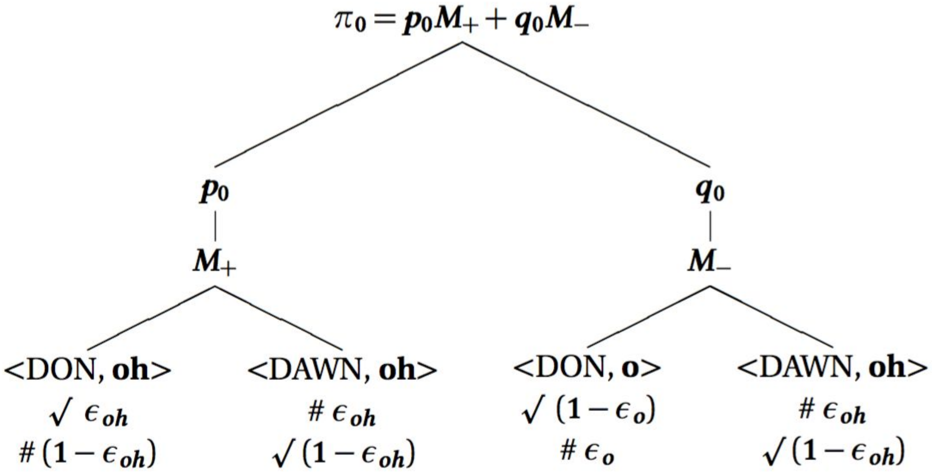

Yang 2009

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- **Asymmetric**
  - **If a listener is unmerged, merged speakers create misunderstandings**
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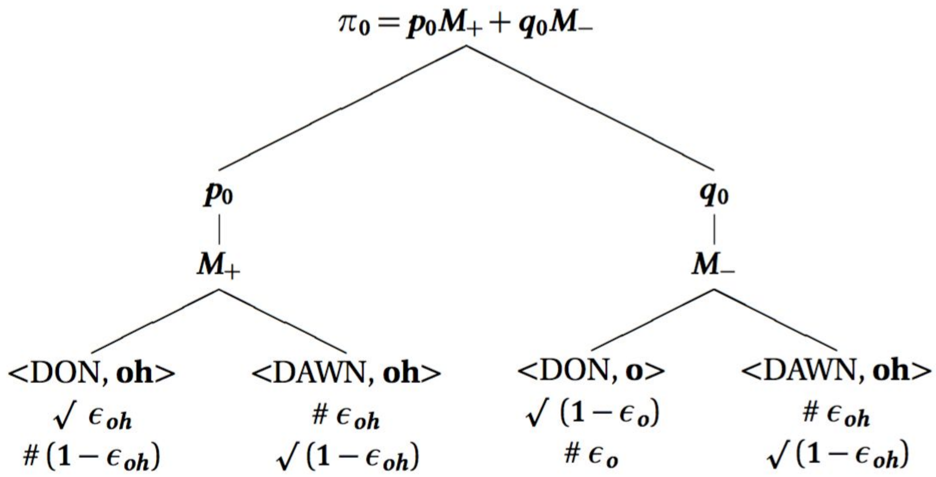
- **Calculated for cot-caught, if at least ~17% of input is merged, the learner acquires the merged grammar**



Yang 2009

The Problem

- Except under incredibly specific network settings, a near-uniform population fixes at 0% or 100% in a couple iterations
  - In our model, alpha must be within a 0.005 window to avoid this
  - alpha is never so finicky otherwise
- Not what has happened empirically

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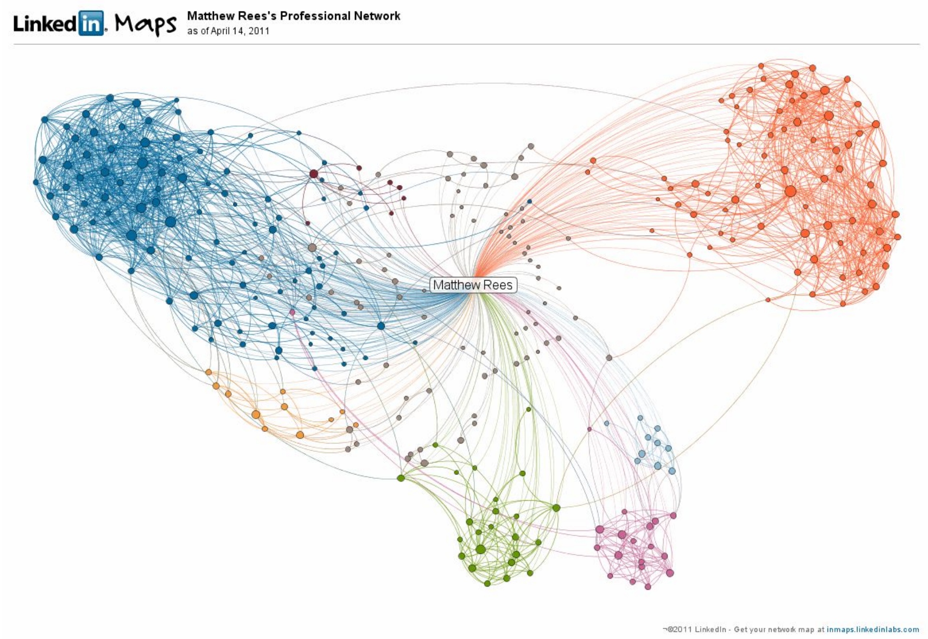
  - Tend to consist of many tight clusters loosely connected together
  - Echos of Milroy & Milroy's "strong and weak connections"



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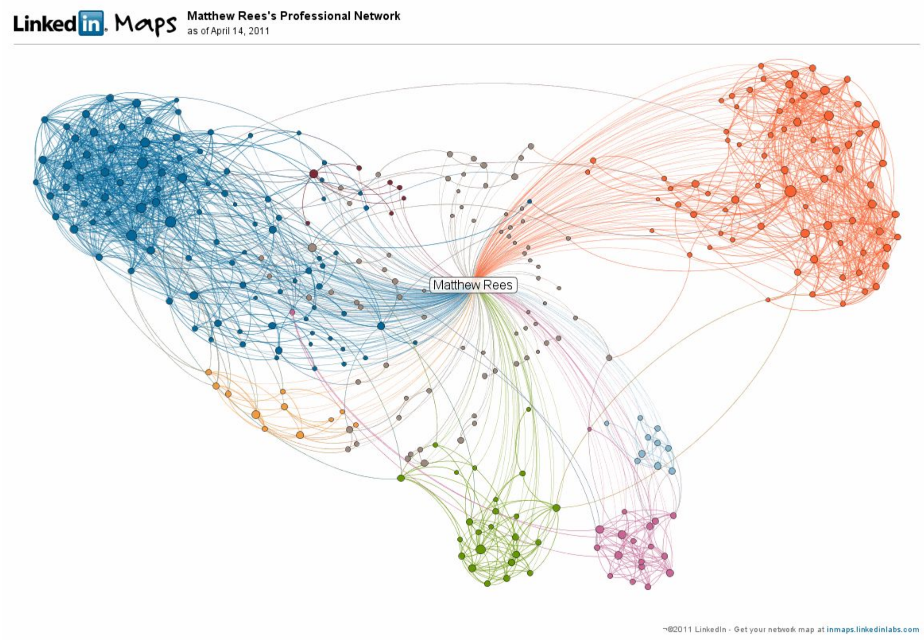

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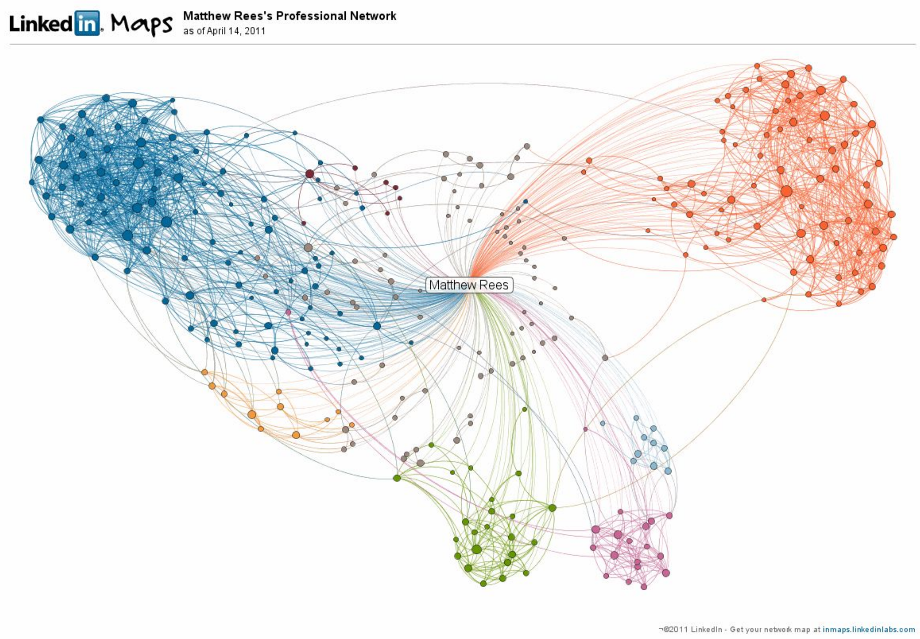
  - etc.
- So we consider a loosely connected network of centralized clusters



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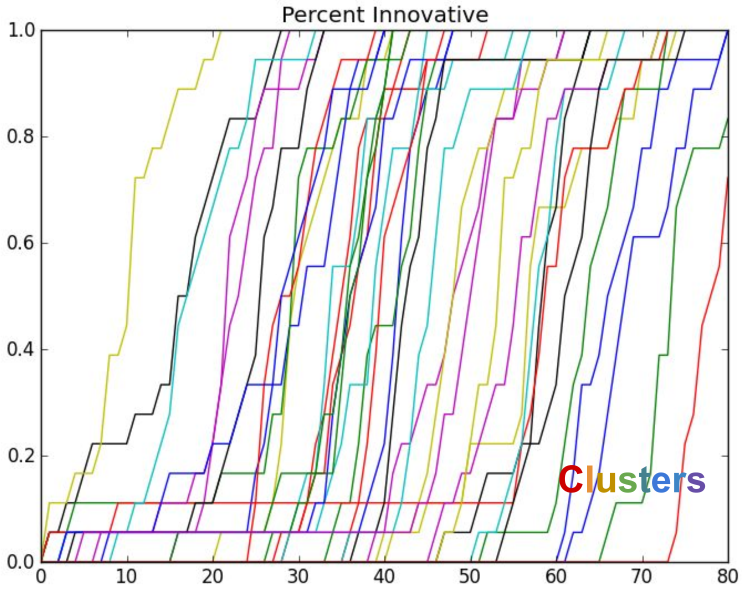

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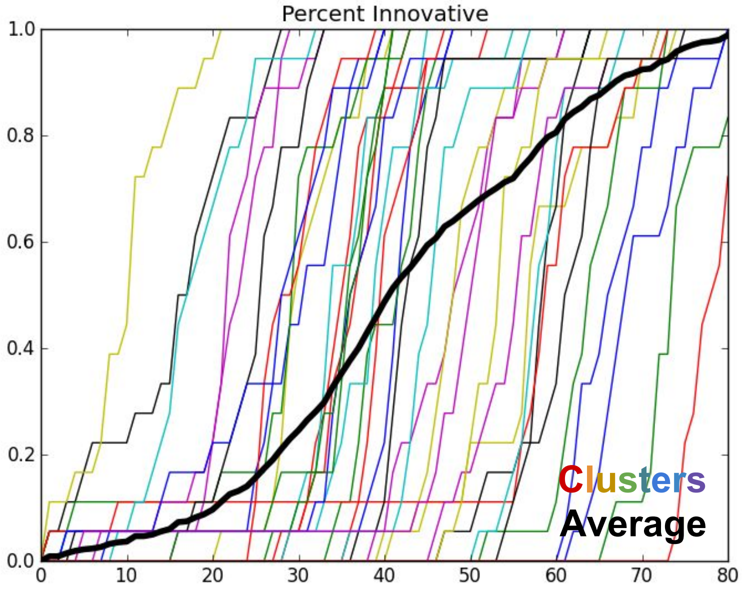
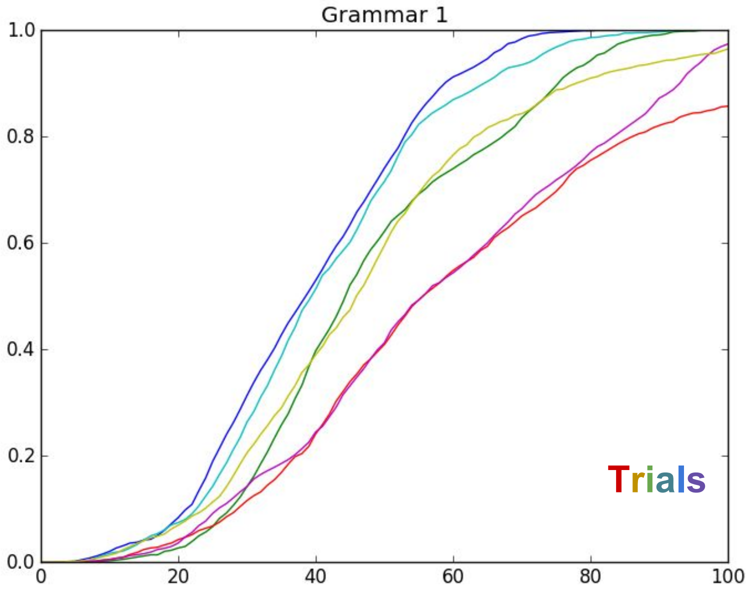
- But the community average is an S-curve



Properties of Change

The averaged S-curve slope:

- **depends on the grammatical advantage *and* the network**



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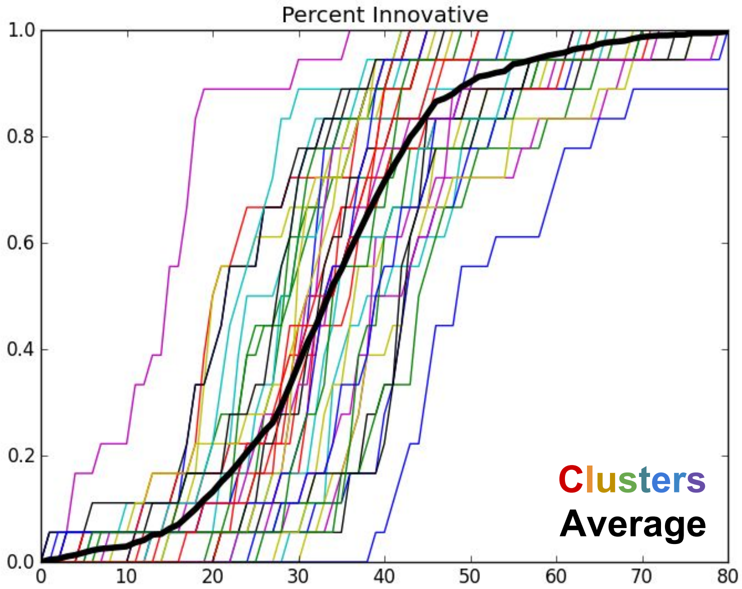

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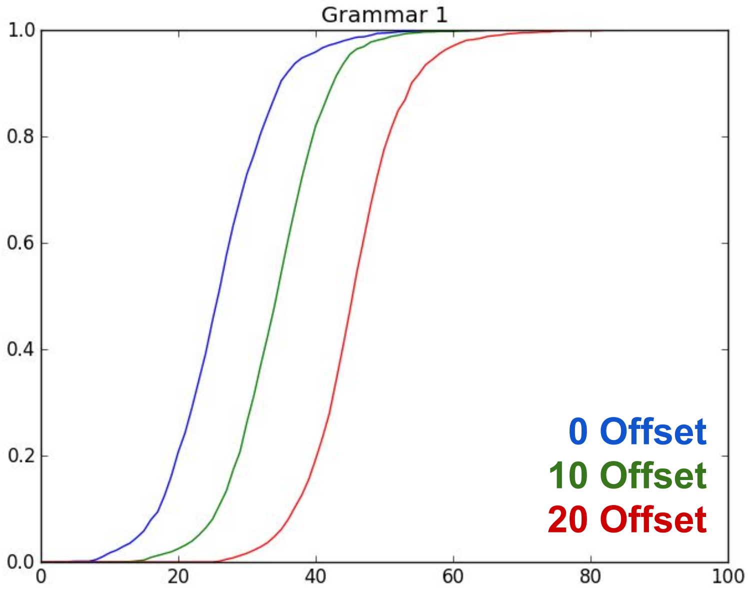

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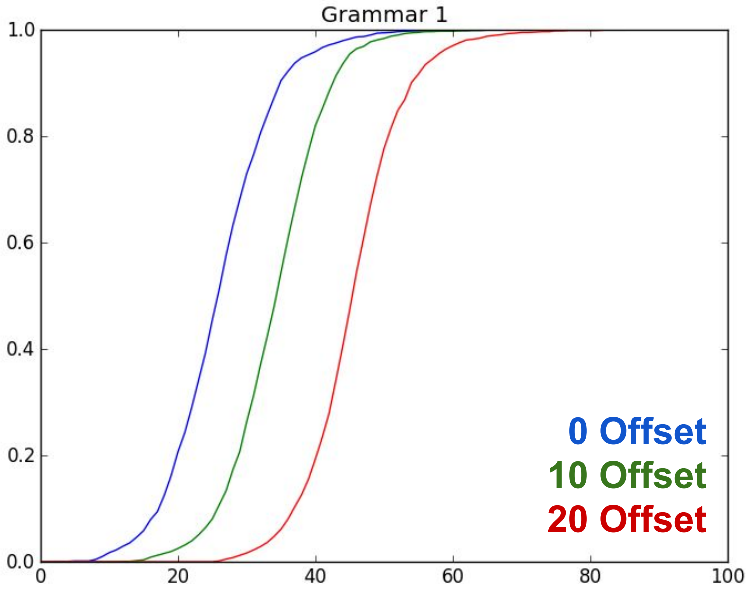
  - **Is compatible with the Constant Rate Effect**



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- S-curve change is possible outside competing-grammars scenarios
- But competing- and single-grammars behave differently on small scales
- Population effects preserve CRE across simultaneous changes with the same advantage

Complex Paths of Change:

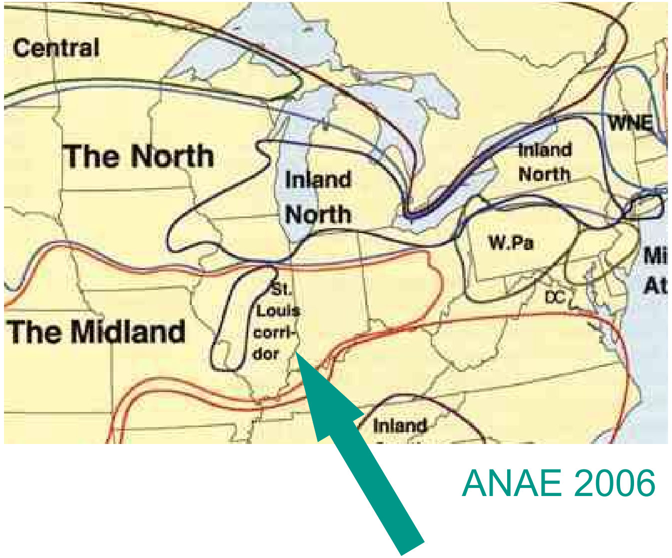
NCS in the St. Louis Corridor

Not all Change is Ideal

- An empirical fact
- Some change does not reach completion
- So it is obviously not S-shaped

The St. Louis Corridor

- Dialect region within US Midlands between Chicago and St. Louis
- But has features from the Inland North
    - Northern Cities Shift (NCS)
    - Has advanced and retreated



ANAE 2006

The St. Louis Corridor

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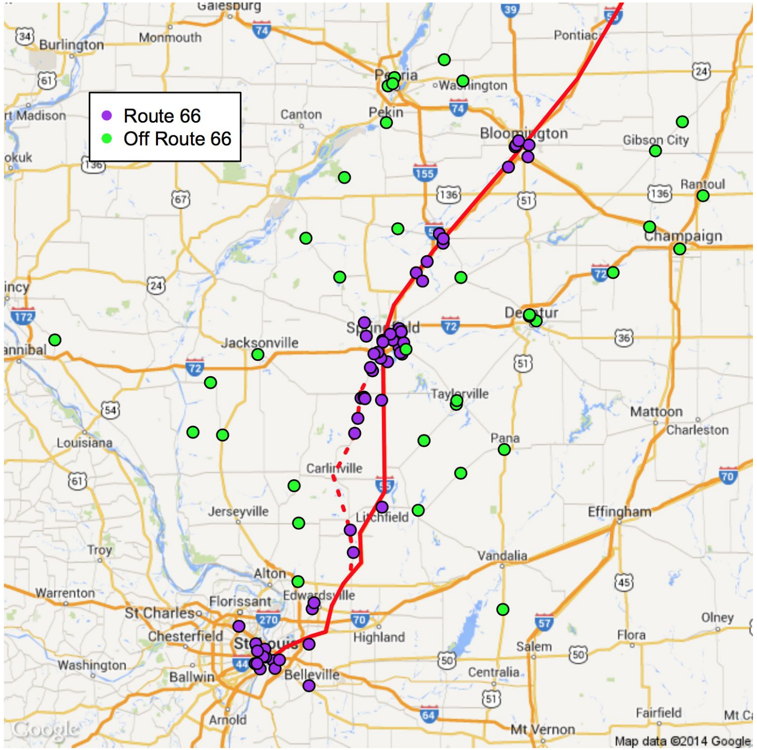

Friedman 2014

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    - NCS peaks first On-Route
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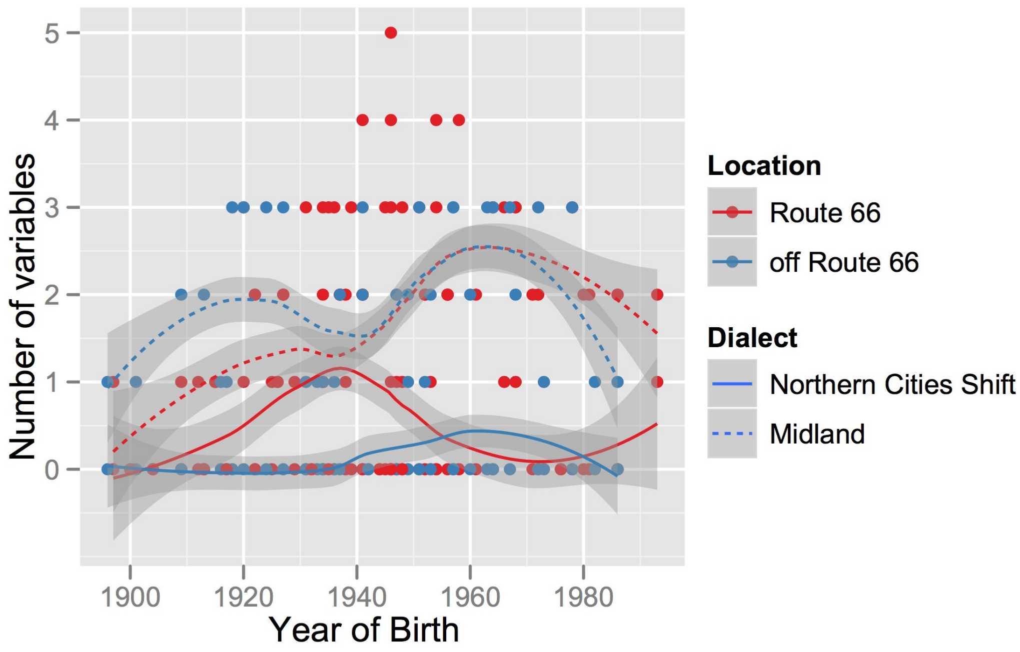

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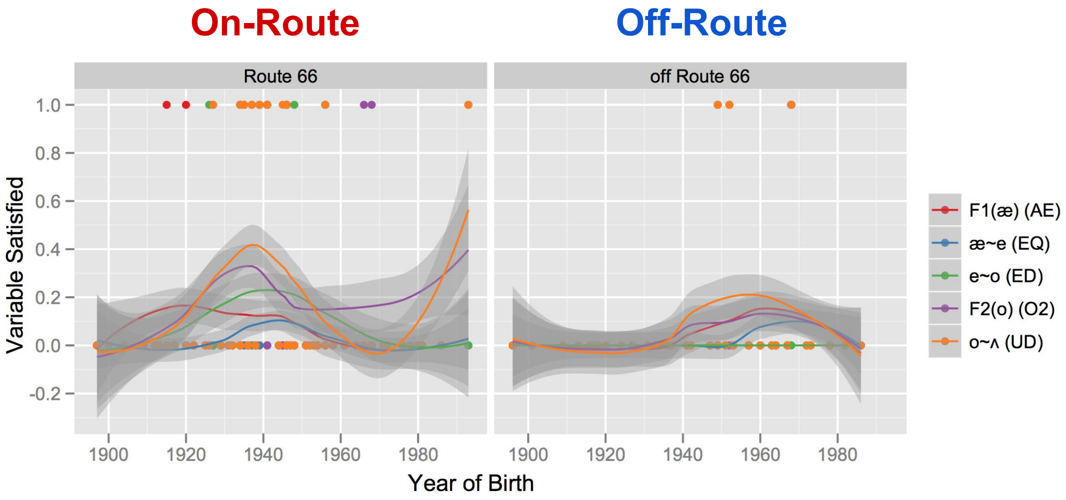

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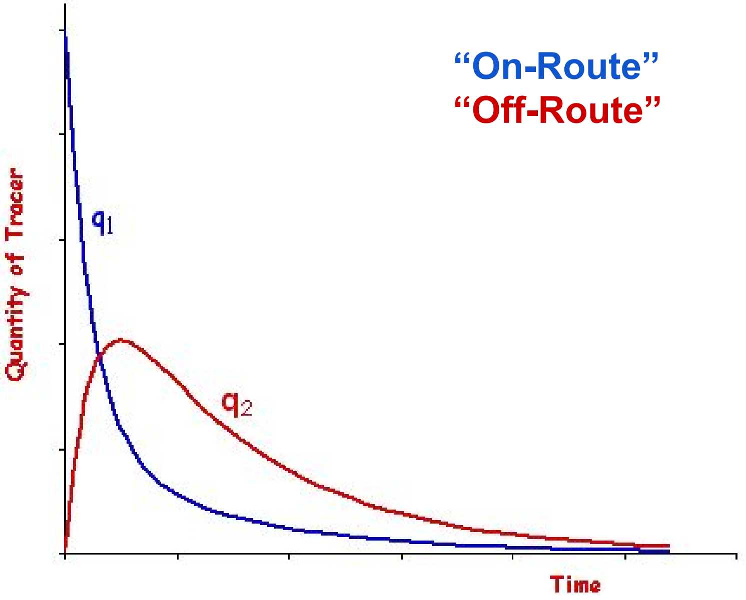
- **Typical of *two-compartment systems***

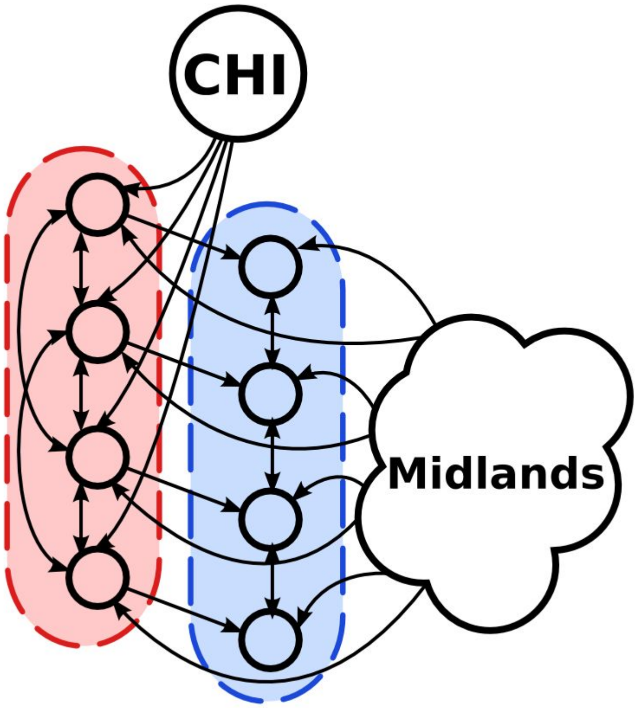


Wikipedia

Modelling the Corridor: Network Structure

Community Types:

- Midlands (1; “background”)
- Chicago (1)
- On-Route (19)
- Off-Route (19)



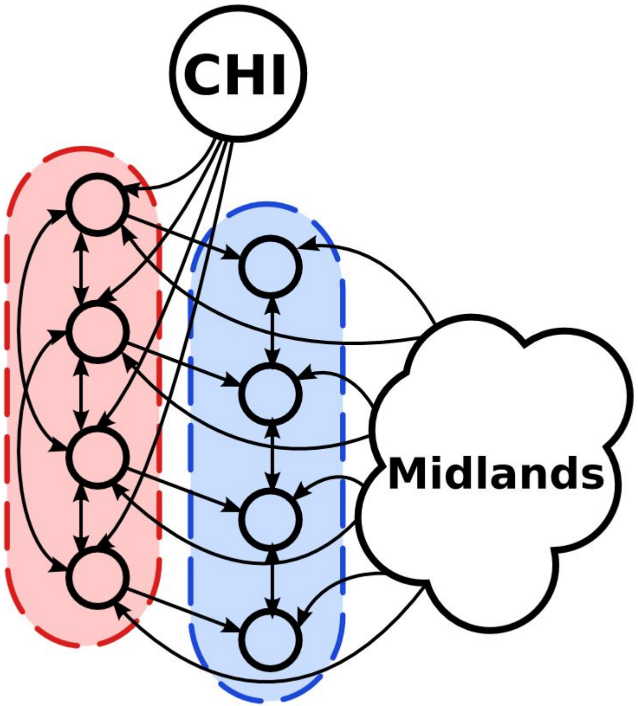
Modelling the Corridor: Network Structure

Community Types:

- Midlands (1; “background”)
- Chicago (1)
- On-Route (19)
- Off-Route (19)

Connections:

- Midlands to all On-Route and Off-Route
- Chicago to all On-Route
- On-Route to two adjacent On-Route
- On-Route to one adjacent Off-Route
- Off-Route to one adjacent Off-Route



Modelling the Corridor: History

- **Vary a single parameter: Direction of movement to On-Route communities**

Modelling the Corridor: History

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    - Strength of connections between On-Route and Off-Route
    - Strength of connections between On/Off-Route and Chicago/Midlands
    - Advantage of NCS
    - Etc.

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  - **Strength of connections between On-Route and Off-Route**
  - **Strength of connections between On/Off-Route and Chicago/Midlands**
  - **Advantage of NCS**
  - **Etc.**
- **And the results would be less meaningful**

Modelling the Corridor: History

- **Vary a single parameter: Direction of movement to On-Route communities**
- **Tests Great Depression hypothesis**

Stage 1 - 5 iterations

No movement (speaker interaction only)

Stage 2 - 20 iterations

2% movement from Chicago to On-Route “Great Depression”

Stage 3 - 75 iterations

2% movement from Midlands to On-Route “Post-Depression”

Modelling the Corridor: The Variable

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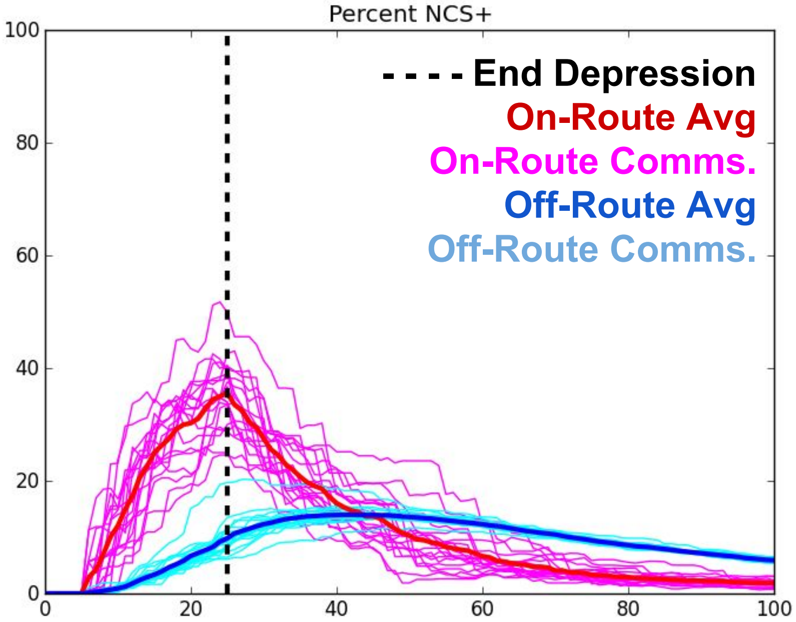
- **Tested as neutral, slightly advantaged, and heavily advantaged change**

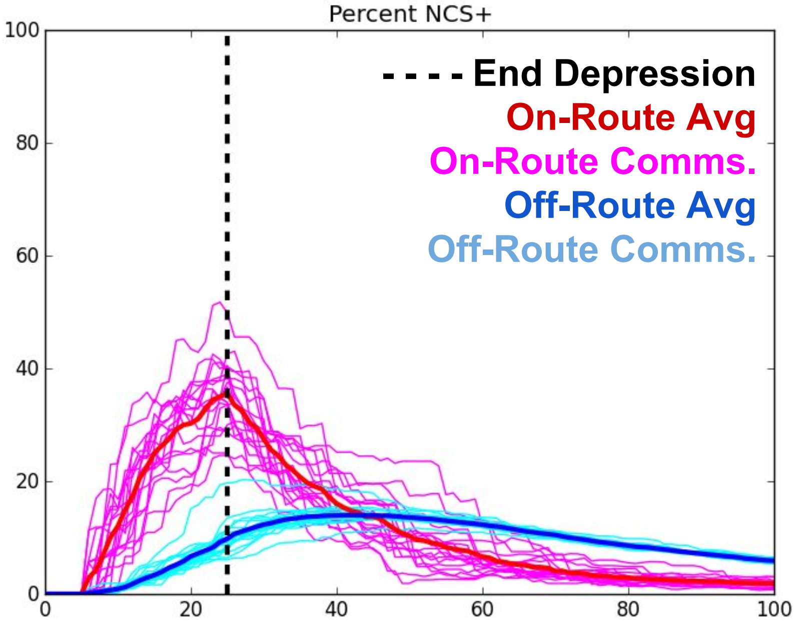
Results: Neutral Change

- **A classic two-compartment pattern arises**



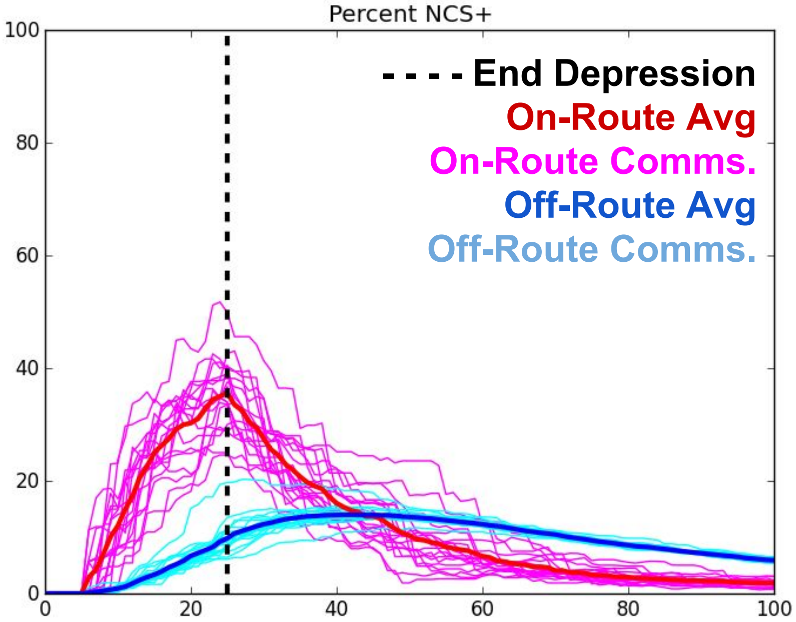
Results: Neutral Change

- A classic two-compartment pattern arises
- NCS peaks higher and earlier On-Route than Off-Route



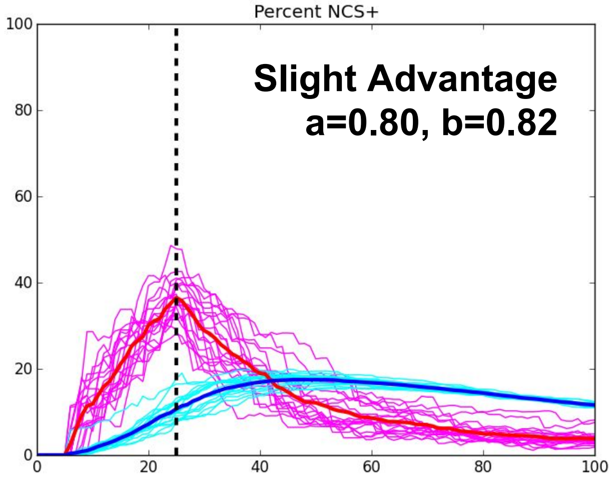
Results: Neutral Change

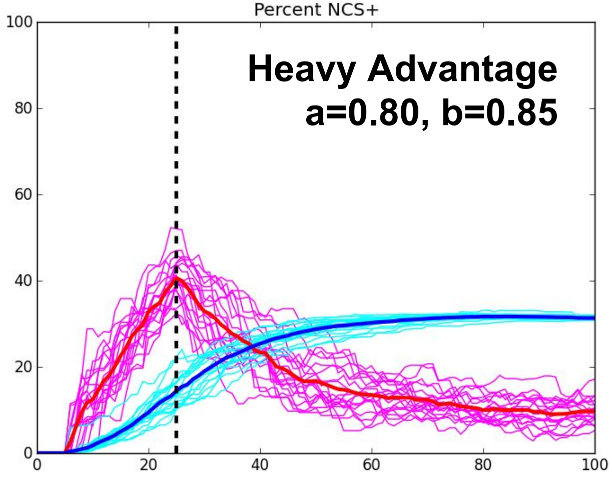
- A classic two-compartment pattern arises
- NCS peaks higher and earlier On-Route than Off-Route
- NCS continues to increase Off-Route even after On-Route population movements are reversed



Results: Advantaged Change

- **Advantaged change resists being "tamped down" Off-Route**
  - **NCS recedes given a slight advantage**
  - **NCS advances given a heavy advantage**





Results: Advantaged Change

- **Advantaged change resists being “tamped down” Off-Route**
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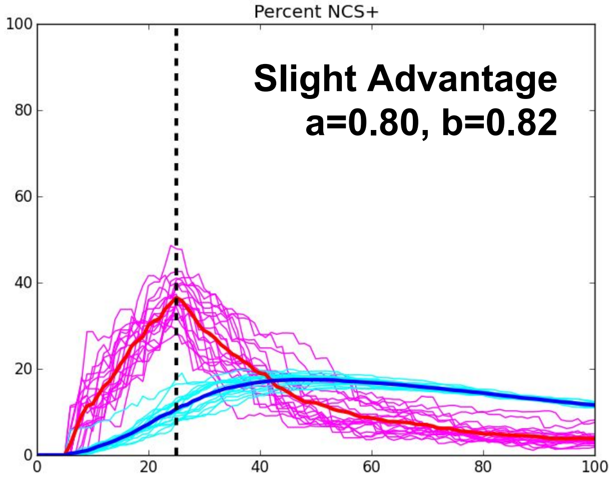
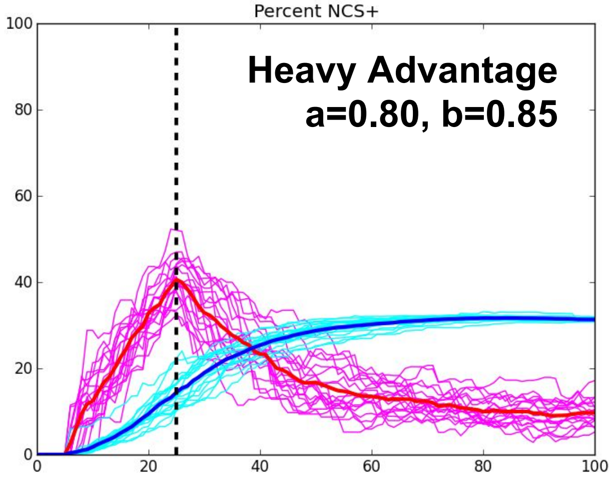
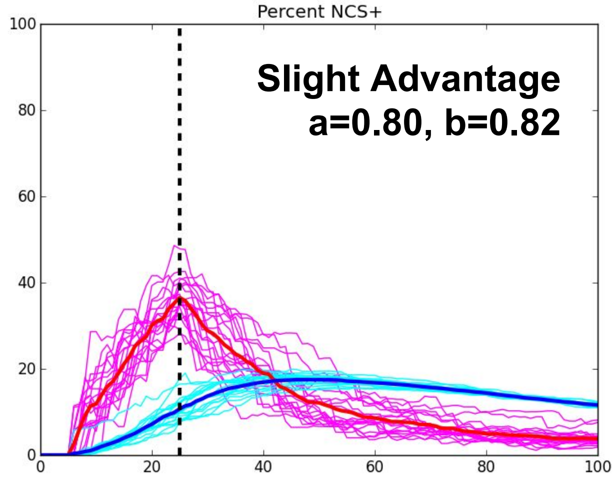
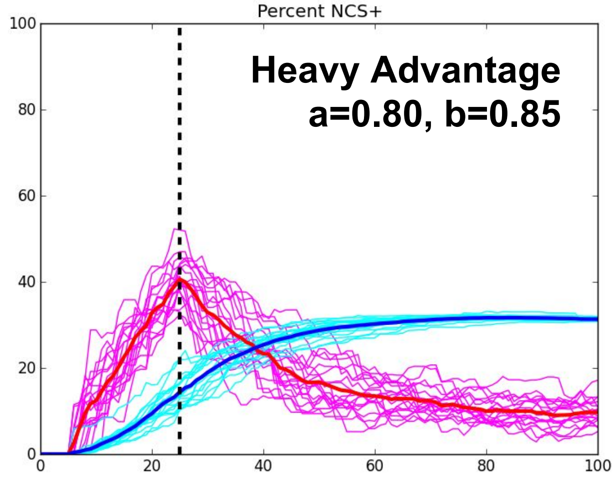




Results: Advantaged Change

- Advantaged change resists being "tamped down" Off-Route
  - NCS recedes given a slight advantage
  - NCS advances given a heavy advantage
- Exists some threshold above which indirect action On-Route is insufficient
- Can be solved with additional model parameters
  - Rate of movement Off-Route
  - The advantage itself
  - etc.





Final Takeaways

Population models and learning models interact!

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    - Under what assumptions are results generalizable?

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Population models and learning models interact!

- **Assumptions must be carefully considered when modelling change**
  - **Under what assumptions are results generalizable?**
- **Attested paths of change are governed by these interactions**
  - **Sometimes explicitly e.g., the St. Louis Corridor**
  - **Sometimes implicitly e.g., New England cot-caught**

End

Code Available here:

github.com/jkodner05/NetworksAndLangChange

Extra slides: Diffusion

Diffusion

$$\mathbf{P}_{t+1} = \mathbf{B}^\top \alpha (\mathbf{I} - (1-\alpha)\mathbf{A})^{-1} \mathbf{H} (\mathbf{H}^\top \mathbf{H})^{-1}$$

- A   *n* x *n* adjacency matrix
- α   jump parameter
- H   *n* x *c* community-membership
- B   *c* x *g* distr. of grammars in comms
- P   *c* x *g* distr. of grammars in inputs

- Indicates directed weighted edges between speakers in network
- Column stochastic
- Easy to make undirected or unweighted

Diffusion

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- **Decides "fluidity" of interactions**
- **Jump distances follow a geometric distribution**
  - **Speakers are most likely to intera adjacent speakers**
  - **But occasionally talk to others far away**
- **Also implemented with Poisson distribution**

Diffusion

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- Indicator matrix
- Defines “community” membership
- More on this later...

Diffusion

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- **Distribution of grammars**
- **According to which community members produce utterances**

Diffusion

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- **Distribution of grammars**
- **Heard by learners of each community**

Tracking Individuals

- **The model can the average behavior of "communities" rather than individuals**
- **If $c = n$, then H is n x n, and the full descriptive detail of the model is available**
  - **H becomes the identity matrix, and the formula for P can be rewritten**

$$\mathbf{P}_{t+1} = \mathbf{B}^\top \alpha \left(\mathbf{I} - (1-\alpha)\mathbf{A}\right)^{-1}$$

Tracking Communities

- **If fine-grain detail is unnecessary, tracking community averages provides substantial computational speedup when $c \ll n$**
- **If each community is internally uniform, n x n A admits a c x c equitable-partition A^{π}**
- **Yielding a more efficient but equivalent update formula for P**

$$\mathbf{A}^{\pi} = (\mathbf{H}^{\top}\mathbf{H})^{-1}\mathbf{H}^{\top}\mathbf{A}\mathbf{H}$$

$$\mathbf{P}_{t+1} = \alpha\mathbf{B}^{\top}\mathbf{H}\left(\mathbf{I} - (1-\alpha)\mathbf{A}^{\pi}\right)^{-1}(\mathbf{H}^{\top}\mathbf{H})^{-1}$$

Anecdotally, I can run n = 20,000 nets on my laptop with A^{π} about as fast as n = 2,000 net with A

Extra Slides: Transmission

Transmission Example

- Let there be two languages L_1 and L_2, the extensions of g_1 and g_2, produced with probabilities P_1 and P_2.
- $a = P_1[L_1 \text{ union } L_2]$ $1 - a = P_1[L_1 \backslash L_2]$
- $b = P_2[L_1 \text{ union } L_2]$ $1 - b = P_2[L_2 \backslash L_1]$

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- $b = P_2[L_1 \text{ union } L_2]$ $1 - b = P_2[L_2 \backslash L_1]$
- Let T_1 and T_2 be transition matrices assuming g_1 and g_2 are the target grammars respectively
- $T_1 = [1 \quad 0 \;\; ; \;\; 1\text{-}a \quad a]$ $T_2 = [b \quad 1\text{-}b \;\; ; \;\; 0 \quad 1]$

Transmission Example

$$T1 = \begin{bmatrix} 1 & 0 \\ 1-a & a \end{bmatrix}$$

$$T2 = \begin{bmatrix} b & 1-b \\ 0 & 1 \end{bmatrix}$$

- **If the target grammar is g1, then in the limit...**

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- **If the target grammar is g1, then in the limit...**
  - **Learners who initially hypothesize g1 will always remain in g1**
  - **Learners who initially hypothesize g2 will remain at g2 with probability a**
  - **Or switch to g1 with probability 1-a**